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ALGEBRAIC KAHN-PRIDDY THEOREM

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There is an epimorphism from the stable homotopy of the infinite real projective space RP^∞ to the (2-component) stable homotopy of spheres. This is the well known Kahn-Priddy theorem and was originally a conjecture of M. E. Mahowald and G. W. Whitehead. Mahowald also conjectured that the epimorphism should occur in the E_2 terms of the Adams spectral sequences. We prove this conjecture is true.

1. Introduction. In his memoir [3] M. E. Mahowald made two conjectures on a specific map λ from the suspension spectrum P^∞ of the infinite real projective space RP^∞ to the sphere spectrum S^0. He conjectured that λ induces epimorphisms in homotopy and in E_2 terms of the mod 2 Adams spectral sequences which are Ext groups over the mod 2 Steenrod algebra A. The first conjecture, which was also conjectured by G. W. Whitehead [5], has been proved by D. S. Kahn and S. B. Priddy [2] and is now known as the Kahn-Priddy theorem. The second conjecture, however, remains unproved. In this paper we record a proof of the truth of Mahowald's conjecture on this "algebraic Kahn-Priddy theorem". The result is stated as Theorem 1.1 below.

The map λ cited above has the property that $\lambda_\#: \pi_1(P^\infty) = Z_2 \to \pi_1(S^0) = Z_2$ is an isomorphism. Kahn and Priddy proved their theorem not just for λ but also for any map $g: P^\infty \to S^0$ which induces isomorphism in $\pi_1(\ )$. We shall state our "algebraic Kahn-Priddy theorem" also for any such map g.

THEOREM 1.1. *For any map $g: P^\infty \to S^0$ that induces an isomorphism in the first stem homotopy groups the induced homomorphism*

$$g_*: \mathrm{Ext}_A^{s,t}(H^*(P^\infty), Z_2) \longrightarrow \mathrm{Ext}_A^{s+1,t+1}(H^*(S^0), Z_2)$$

is an epimorphism for all s and t with $t - s \geqq 1$.

Here $H^*(\ )$ is the reduced mod 2 cohomology functor. The result in Theorem 1.1 is what Mahowald had conjectured in [3] for $g = \lambda$. We shall see that only the isomorphism $g_\#: \pi_1(P^\infty) \to \pi_1(S^0)$ is relevent. The map λ does not play a special role.

I would like to thank Professor Mahowald for encouragement to prove his conjecture.

2. Proof. The main result used to prove Theorem 1.1 is