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SOME TOPOLOGICAL PROPERTIES OF SPACES OF MEASURES

G. KOUMOULLIS

Let X be a completely regular space and $M_\tau(X)$, $M_t(X)$ and $M_c(X)$ the spaces of the τ-additive measures, tight measures and measures with compact support on X, endowed with the weak topology. The aim of this paper is to study topological properties that devolve from X to $M_\tau(X)$, $M_t(X)$ and $M_c(X)$ or their positive cones $M_\tau^+(X)$, $M_t^+(X)$ and $M_c^+(X)$. It is proved that if X is paracompact (resp. Lindelöf) and Cech complete, then $M_\tau^+(X)$ and $M_t^+(X)$ have the same properties, but $M_c^+(X)$ does not (unless X is compact). If X is realcompact then $M_c(X)$ has the same property, but $M_\tau(X)$ and $M_t(X)$ need not. However, if X is realcompact paracompact, then $M_\tau(X)$ is realcompact.

Let X be a completely regular space and $C(X)$ the space of bounded real-valued continuous functions on X with the supremum norm. The spaces $M_\sigma(X)$, $M_\tau(X)$, $M_t(X)$ and $M_c(X)$ of measures on X are defined as subsets of the dual of $C(X)$ ([22]) and can be described using the Stone-Cech compactification of X ([13]). Identifying each point x of X with the point mass ε_x, X can be considered as a closed subset of $M_s(X)$ and $M_s^+(X)$ (for $s = \tau$, t or c) endowed with the weak topology which is defined by $C(X)$. Our purpose is to find topological properties that devolve from X to $M_s(X)$ or $M_s^+(X)$.

Notations and preliminary results are given in § 1. In § 2 it is proved that certain topological properties which can be described by perfect functions with values in metric spaces, devolve from X to $M_s^+(X)$ but not to $M_s(X)$. For this purpose it is shown that a perfect function between two spaces X and Y induces a perfect function between $M_s^+(X)$ and $M_s^+(Y)$. Section 3 is concerned with the property of realcompactness. Using a result of Corson, it is proved that $M_s(X)$ is realcompact if and only if every σ-additive measure on X which on every countably generated σ-algebra of Baire sets coincides with some element of $M_s(X)$ is, in fact, an element of $M_s(X)$. Realcompactness devolves from X to $M_c(X)$, but not to $M_\tau(X)$ and $M_t(X)$.

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1. Preliminaries and notations.

A basic reference for the