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ON WEAK RESTRICTED ESTIMATES AND ENDPOINT PROBLEMS FOR CONVOLUTIONS WITH OSCILLATING KERNELS (I)

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Throughout we consider $K(t) = e^{i|t|^a}/|t|^b$, $a > 0$, $a \neq 1$, $b < 1$ and $t \in \boldsymbol{R}$. Here we consider for fixed $\lambda, \mu > 0$ the function $B(\lambda, \mu; K) = B(\lambda, \mu) = \sup_{\chi_\lambda \chi_\mu} \int \chi_\lambda(x) K * \chi_\mu(x) dx$ where the sup is taken over all "characteristic" functions χ_λ, χ_μ with complex signs (i.e., χ_μ is a measurable function for which $|\chi_\mu| = 1$ on E, $|\chi_\mu| = 0$ off E and $|E| \leqq \mu$ $(\mu > 0)$). We estimate $B(\lambda, \mu; K)$ within constant factors from above and below. This settles the endpoint problems for these kernels, at least in the weak restricted sense.

0. Introduction. This paper is concerned with (L_p, L_q)-mapping properties of the operator

$$g = K*f \ , \qquad g(x) = \int K(x-y)f(y)dy \qquad (x, y \in \boldsymbol{R}^n) \ ,$$

in particular with (weak restricted) estimates

$$(1) \qquad \left|\int \chi_\lambda(K*\chi_\mu)\right| \leqq c_{pq}\lambda^{1/q'}\mu^{1/p}(1/q + 1/q' = 1) \ ,$$

where, e.g., χ_μ denotes a "characteristic" function with complex signs, i.e., a measurable function with $|\chi_\mu| = 1$ on E, $|\chi_\mu| = 0$ off E, $|E| \leqq \mu(\mu > 0)$. Let us denote by $B(\lambda, \mu) \equiv B(\lambda, \mu; K)$ the quantity

$$\sup_{\chi_\lambda \cdot \chi_\mu} \left|\int \chi_\lambda(K*\chi_\mu)\right| = \sup_{\chi_\lambda \cdot \chi_\mu} \left|\iint K(x+y)\chi_\lambda(x)\chi_\mu(y)dxdy\right| \ ,$$

where the sup varies over all characteristic functions χ_λ, χ_μ with fixed $\lambda > 0$, $\mu > 0$. Our present problem will be to estimate $B(\lambda, \mu)$ as closely as possible from above and below.

In earlier papers [4], [9], [13], [14] we already discussed the mapping properties for oscillating kernels. In [4] we gave, in part, the mapping properties for the kernels

$$(2) \qquad K(t) = \frac{e^{i|t|^a}}{|t|^b}(0 \neq t \in \boldsymbol{R}) \text{ with } a > 0,\ a \neq 1,\ b < 1$$

except for the endpoints. By means of the function $B(\lambda, \mu)$ we settle the endpoint problems in the weak restricted sense. Furthermore, we determine $B(\lambda, \mu)$ within constant factors from above and