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EXISTENCE OF STRONG SOLUTIONS FOR STOCHASTIC DIFFERENTIAL EQUATIONS IN THE PLANE

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Let B be the 2-parameter Brownian motion on $D = [0, \infty] \times [0, \infty)$ and Z be a 2-parameter stochastic process defined on the boundary ∂D of D. Consider the non-Markovian stochastic differential system in 2-parameter

$$\begin{cases} dX(s, t) = \alpha(s, t, X)dB(s, t) + \beta(s, t, X)dsdt & \text{for} \quad (s, t) \in D \, , \\ X(s, t) = Z(s, t) & \text{for} \quad (s, t) \in \partial D \, . \end{cases}$$

Under the assumption that the coefficients α and β satisfy a Lipschitz condition and a growth condition and the assumption that Z has continuous sample functions and locally bounded second moment on ∂D, it is shown in this paper that the differential system has a strong solution. Pathwise uniqueness of solution is established under the assumption of the Lipschitz condition.

0. Introduction. Recently several papers on stochastic integrals in the plane have appeared (see [2], [3], [9], [11] and [13]). In the present paper we treat stochastic differential equations in the plane. The domain of definition of the stochastic integrals and stochastic differential equations we consider is the positive quadrant $D = [0, \infty) \times [0, \infty)$ in which a partial order $(s, t) \prec (u, v)$ for $s \leqq u$ and $t \leqq v$ is introduced. The object of our study is a stochastic differential equation of the type

$$dX(s, t) = \alpha(s, t, X)dB(s, t) + \beta(s, t, X)dsdt \, ,$$

or, to be precise,

$$\begin{aligned} (0.1) \qquad & X(s, t) - X(s, 0) - X(0, t) + X(0, 0) \\ & = \int_{[0,s]\times[0,t]} \alpha(u, v, X)dB(u, v) \int_{[0,s]\times[0,t]} \beta(u, v, X)d(u, v) \, , \end{aligned}$$

where B is a 2-parameter Brownian motion on a probability space $(\Omega, \mathfrak{F}, P)$ and the domain D. The precise defininions of B and the two stochastic integrals appearing on the right side of (0.1) are given in § 2. The case in which the coefficients of the stochastic differential equation depend on X only to the extent that they depend on $X(s, t)$, i.e., the stochastic differential equation

$$dX(s, t) = a(X(s, t))dB(s, t) + b(X(s, t))dsdt$$

was treated by Cairoli [2]. The coefficients α and β in our equation