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# ON COMPACTLY PACKED RINGS

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**A commutative ring with identity is *compactly packed by primes* (briefly: a C. P.-ring) if whenever an ideal $I$ of $R$ is contained in the union of a family of prime ideals of $R$, then $I$ is actually contained in one of the primes of the family. The aim of this note is to characterize Noetherian P. C.-rings ideal theoretically and, with suitable restrictions, in terms of Picard groups.**

The notion of a C. P.-ring was introduced by C. Reis and T. Viswanathan in [6], where Noetherian C. P.-rings were characterized by the property that primes are radicals of principal ideals, and it was asked if it suffices to have maximal ideals be radicals of principal ideals. N. Popescu studied the torsion theoretic aspects of C. P.-rings in [4], where he extended the preceding result to semi-noetherian rings and asked if every semi-noetherian C. P.-ring must have Krull dimension at most one. Independently, W. Smith [8] characterized all C. P.-rings as those rings for which every prime ideal is the radical of a principal ideal. In this note we answer the preceding questions (the former in the affirmative and the latter in the negative) and examine a connection between torsion Picard groups and the C. P.-property.

We would like to express our thanks to William Krauter and Roger Wiegand for several enlightening observations regarding the connection between Picard groups and C. P.-rings.

As usual Spec($R$) denotes the set of prime ideals of $R$, $V(I) = \{P \in \mathrm{Spec}(R) \mid I \subseteqq P\}$ for subsets $I$ of $R$ and $D(I)$ denotes the complement of $V(I)$. Denote the radical of the ideal $I$ by rad($I$) and the residual $\{r \in R \mid rI \subseteqq J\}$ by $(J{:}\, I)$. As we have noted, the first equivalence of the following theorem is due to Smith [8]; we supply a proof for convenience of the reader.

THEOREM 1. *The ring $R$ is a* C. P.*-ring if and only if every prime ideal is the radical of a principal ideal, in which case every radical ideal is the radical of a principal ideal. Moreover the Noetherian ring $R$ is a* C. P.*-ring if and only if every maximal ideal is the radical of a principal ideal.*

*Proof.* Suppose the $R$ is a C. P.-ring and $I$ is a radical ideal of $R$, i.e., $I = \mathrm{rad}(I)$. The C. P. property implies that $I \not\subseteqq \cup D(I)$, whence there exists an element $x \in I$ such that $V(x) \subseteqq V(I)$. But the converse