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TOPOLOGICAL PROOF OF THE G-SIGNATURE THEOREM FOR G FINITE

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The G-Signature Theorem was originally proved by Atiyah and Singer as a corollary of their general index theorem for elliptic operators. Subsequently Ossa gave a proof for G finite and the fix point set orientable. His methods are mainly topological. However, he uses the theory of elliptic operators to show the g-signature of a fix point free diffeomorphism of finite order is zero. Janich and Ossa gave a short completely topological proof of the theorem for involutions. In part one, we give a complete proof for semi-free actions and simultaneously a proof for general actions modulo the theorem for fix point free actions. In essence our argument here is similar to that of Ossa. However it is shorter and conceptually simpler. Also we derive the formula in a natural way as opposed to verifying it. In part two, we prove a theorem which we use in part one to prove the result for fix point free actions. **I wish to thank my advisor Professor E. Thomas for much help and encouragement.**

Part One. By considering the cyclic subgroup generated by a given element, we may restrict our attention to cyclic group actions. By a $\boldsymbol{Z}_d$ manifold (g, M) we will mean a smooth, oriented, compact manifold (without boundary unless otherwise stated or obvious) together with an orientation preserving diffeomorphism of order d. Frequently, we will omit the g in referring to $\boldsymbol{Z}_d$ manifolds M. We denote the disjoint union by $+$, the disjoint union of r copies of M by rM, and the disjoint difference (one reverses the orientation of the second manifold) by $-$. A $\boldsymbol{Z}_d$ manifold M bounds (resp. rationally bounds) if M (resp. $r \cdot M$) is the boundary of a $\boldsymbol{Z}_d$ manifold. Two $\boldsymbol{Z}_d$ manifolds are bordant (resp. rationally bordant) if their disjoint difference bounds (resp. rationally bounds). The collection of bordism classes of $\boldsymbol{Z}_d$ manifolds forms a graded ring $O_*(\boldsymbol{Z}_d)$ in the usual manner.

Let (g, B) denote the d-fold branched cover of S^2 along d points together with the deck translation which rotates neighborhoods of the fixed points through an angle $2\pi/d$. Let (g, P_n) denote the action on n-dimensional complex projective space given by $g[z_0, \cdots, z_{n-1}, z_n] = [z_0, \cdots, z_{n-1}, \omega z_n]$. Here and throughout part one $\omega = \exp(2\pi i/d)$. The fix point set has two components: the point $[0, \cdots, 0, 1]$ where the action on the trivial normal bundle is multiplication by $\bar{\omega}$, and