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# AUTOMORPHISMS OF DIMENSION GROUPS AND THE CONSTRUCTION OF AF ALGEBRAS

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**Recent results of Edward G. Effros and the author show that if a dimension group is simple, totally ordered and with underlying group $Z^n$, then we can construct explicitly an AF $C^*$-algebra with the given group as its $K_0$ by using the Jacobi-Perron algorithm. While the Jacobi-Perron algorithm breaks down for nontotally ordered groups, we study the construction problem via the consideration of automorphisms of the dimension group. We find the necessary and sufficient condition for a nontotally ordered simple dimension group $(Z^3, P_{(1,\alpha,\beta)})$ being stationary is that both $\alpha$ and $\beta$ lie in the same quadratic number field. We also provide an explicit method for constructing Bratteli diagrams (and hence corresponding $AF$ $C^*$-algebras) for this type of groups.**

Introduction. Since George Elliott introduced dimension theory for approximately finite $C^*$-algebras, considerable progress has been made in the study of AF $C^*$-algebras ([3], [4], [5], [6], [7], [8], [2]). In [5] and [6], Effros and the author raised the question of constructing AF algebras with given dimension groups as their $K_0$, and answered it in the case when the given dimension group is simple totally ordered and with underlying group $Z^n$ by using the Jacobi-Perron algorithm. Based on this and some examples of nontotally ordered simple dimension groups ([6, §4]). we conjectured for any simple dimension group $G$ with underlying group $Z^n$, both that there exists an inductive sequence

$$Z^n \xrightarrow{\varphi_1} Z^n \xrightarrow{\varphi_2} Z^n \longrightarrow \cdots$$

where $\varphi_k \in \mathrm{GL}(n, Z)$ with nonnegative entries such that $\varinjlim (Z^n, \varphi_k) \cong G$ and that there exists effective methods for constructing these $\varphi_k$'s. In the meantime, our results have been applied by Cuntz, Krieger, Pimsner and Voiculescu ([2], [11]) to problems in topological Markov chains and to the irrational rotation $C^*$-algebras.

While the recent work of Riedel [12] supports the first part of conjecture, the construction problem still remains. Some dimension groups which are certainly worth first consideration are those having unique state (see [4] for definition). In this direction, as motivated by the work of Cuntz and Krieger, we ask the following question about existence and construction: