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ON MEASURABLE PROJECTIONS IN BANACH SPACES

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Let E be a Banach space that is complemented in its bidual by a projection $P: E^{**} \to E$. It is shown that E has the Radon Nikodym property if and only if for every Radon probability measure λ on the unit ball K of E^{**} such that $\omega^* - \int_A x^{**} d\lambda \in E$ for every weak* Borel subset A of K, the projection P is λ-Lusin measurable and for every x^* in E^* the map x^*P satisfies the barycentric formula for λ on K.

J. J. Uhl Jr. asked the following question: Let E be a Banach space which is complemented in its bidual by a projection $P: E^{**} \to E$ which is weak* to norm universally Lusin measurable. Does E have the Radom-Nikodym property?

In [4] we showed that if E is the dual of a Banach space Y and if P is the natural projection from $E^{**} = Y^{***}$ to $Y^* = E$ then the above condition is necessary and sufficient for E to have the Radon-Nikodym property.

In [4] we also showed that for any Banach space E, if P is weak* to weak Baire-1 function then E has the Radon-Nikodym property.

Recently G. Edgar showed using an idea of Talagrand and Weizsäcker that the projection

$$L_1[0, 1]^{**} \longrightarrow L_1[0, 1]$$

is weak* to weak universally-Lusin measurable. This shows that Uhl's question does not have a positive answer in general, however if one examines the results of [4] he can see that if P is Baire-1, it is universally Lusin-measurable and for every x^* in E^* the map x^*P satisfies the barycentric formula. It turns out that a Banach space E has the Radon-Nikodym property if and only if for every Radon probability measure λ on the unit ball K of E^{**} such that $\omega^* - \int_A x^{**} d\lambda \in E$ for every ω^*-Borel subset A of K the projection P is λ-Lusin measurable and for every x^* in E^* the map x^*P satisfies the barycentric formula for λ on K.

Let us fix some terminology and conventions. All topological spaces in this paper will be completely regular. The set of all Radon probability measures on a topological space (X, τ) will be denoted by $M_+^1(X, \tau)$.

DEFINITION 1. Let (X, τ_1) and (Y, τ_2) be two topological spaces and let