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BAER RINGS AND QUASI-CONTINUOUS RINGS HAVE A MDSN

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The notion of a direct summand of a ring containing the set of nilpotents in some "dense" way has been considered by Y. Utumi, L. Jeremy, C. Faith, and G. F. Birkenmeier. Several types of rings including right self-injective rings, commutative FPF rings, and rings which are a direct sum of indecomposable right ideals have been shown to have a MDSN (i.e., the minimal direct summand containing the nilpotent elements). In this paper, the class of rings which have a MDSN is enlarged to include quasi-Baer rings and right quasi-continuous rings. Also, several known results are generalized. Specifically, the following results are proved: (Theorem 3) Let R be a ring in which each right annihilator of a reduced (i.e., no nonzero nilpotent elements) right ideal is essential in an idempotent generated right ideal. Then $R = A \oplus B$ where B is the MDSN and an essential extension of N_t (i.e., the ideal generated by the nilpotent elements of index two), and A is a reduced right ideal of R which is also an abelian Baer ring. (Corollary 6) Let R be an AW^*-algebra. Then $R = A \oplus B$ where A is a commutative AW^*-algebra, and B is the MDSN of R and B is an AW^*-algebra which is a rational extension of N_t. Furthermore, A contains all reduced ideals of R. (Theorem 12) Let R be a ring such that each reduced right ideal is essential in an idempotent generated right ideal. Then $R = A \oplus B$ where B is the densely nil MDSN, and A is both a reduced quasi-continuous right ideal of R and a right quasi-continuous abelian Baer ring.

From [8 & 14], a ring R is (*quasi-*) *Baer* if it has unity and the right annihilator of every (right ideal) nonempty subset of R is generated by an idempotent. A Baer ring is *abelian* if all its idempotents are central. The following examples will give some indication of the wide application of these rings: (i) von Neumann algebras, such as the algebra of all bounded operators on a Hilbert space, are Baer rings [2, pp. 21 & 24]; (ii) the commutative C^*-algebra $C(T)$ of continuous complex valued functions on a Stonian space is a Baer ring [2, p. 40]; (iii) the ring of all endomorphisms of an abelian group G with $G = D \oplus E$, where $D \neq 0$ is torsion-free divisible and E is elementary, is a Baer ring [16]; (iv) any right self-injective von Neumann regular ring is Baer [17, p. 253]; (v) any prime ring is quasi-Baer; (vi) since a $n \times n$ matrix ring over a