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APPROXIMATING COMPACT SETS IN NORMED LINEAR SPACES

JACK GIROLO

It is shown that in normed linear spaces compact sets can be approximated by compact absolute neighborhood retracts in the following sense: If X is a compact subset of a normed linear space, then for every $\varepsilon > 0$ there exists a compact absolute neighborhood retract that contains X and has the property that each point of the retract is within ε of X. If the choice of ε is sufficiently large, the retract can be chosen to be an absolute retract.

Suppose that X is a compact subset of a Banach space B. Then the closure of the convex hull of X, $\overline{\text{conv}(X)}$, is a compact absolute retract that contains X. Browder [4] has shown that if U is an open subset of B that contains X, then there exists a compact absolute neighborhood retract R^* such that $X \subseteqq R^* \subseteqq U$. Both of these results have proven to be useful in Fixed Point Theory. See, for example, the work of Browder mentioned above and the work of Górniewicz and Granas [9].

Let X be a compact subset of a normed linear space N. The purpose of this paper, Theorem 1, is to show that there exists a compact absolute retract R such that $X \subseteqq R \subseteqq N$. Further, it is shown that if U is an open subset of N that contains X, then there exists a compact absolute neighborhood retract R^* such that $X \subseteqq R^* \subseteqq U$.

1. Preliminaries. Absolute retracts and absolute neighborhood retracts for metric spaces will be denoted by AR and ANR respectively. We use the notation $d(x, E)(d(x, y))$ for the distance from a point x to a set E (to a point y). A continuous function $f: X \rightarrow R$ will be called a retraction if $R \subseteqq X$ and $f(x) = x$ for each $x \in R$.

LEMMA 1. *Let $(N, \| \ \|)$ be an infinite dimensional normed linear space, X be a compact subset of N, F be a finite dimensional subspace that is disjoint from X, and ε be greater than 0. Then there exists a finite dimensional subspace E that contains F, is disjoint from X, and for all $x \in X$, $d(x, E) < \varepsilon$.*

Proof Let U_* be an open subset of N. We show that there exists a finite dimensional subspace E_* that contains F, meets U, and is disjoint from X. Let B be the closure of an open set that is contained in U and is disjoint from X. For each $b \in B$, let E_b be the