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A CHARACTERIZATION OF M-IDEALS IN $B(\ell_p)$ FOR $1 < p < \infty$

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For $1 < p < \infty$ the only nontrivial M-ideal in $B(\ell_p)$, the bounded linear operators on ℓ_p, is $K(\ell_p)$, the ideal of compact operators on ℓ_p.

1. Introduction. Certain theorems for $B(H)$ (the bounded linear operators on H a separable Hilbert space) are known to hold for $B(\ell_p)$, $1 < p < \infty$. For example, it is well known that the only nontrivial closed two-sided ideal in $B(\ell_p)$, $1 \leqq p < \infty$ is $K(\ell_p)$, the compact linear operators on ℓ_p. Hennefeld [4] has shown that $K(\ell_p)$ is an M-ideal in $B(\ell_p)$ for $1 < p < \infty$. It is also known that $K(\ell_2)$ is the only nontrivial M-ideal in $B(\ell_2)$. This follows from the fact that in a B^*-algebra, the M-ideals are precisely the closed two-sided ideals [5]. The purpose of this paper is to show that this result also generalizes to $B(\ell_p)$, for $1 < p < \infty$. As this paper is largely based on the work of Smith and Ward [5] it is perhaps not surprising that a result of theirs, namely that every nontrivial M-ideal in $B(\ell_p)$ for $1 < p < \infty$ contains $K(\ell_p)$, has a new proof.

2. Preliminaries. A closed subspace L of a Banach space X is said to be an L-ideal [M-summand] if there exists a closed subspace L' such that $X = L \oplus L'$ and $\|\ell + \ell'\| = \|\ell\| + \|\ell'\|$ [$\|\ell + \ell'\| = \max\{\|\ell\|, \|\ell'\|\}$] for every $\ell \in L$ and $\ell' \in L'$. A closed subspace M of a Banach space X is an M-ideal if $M^\perp$ is an L-ideal in X^*. Note that M-summands are M-ideals, but the latter is a more general concept. [For example, $K(\ell_p)$ is an M-ideal in $B(\ell_p)$ but not an M-summand, as $K(\ell_p)$ is not complemented in $B(\ell_p)$.] For basic properties of M-ideals, L-ideals and M-summands, refer to [1].

The state space S of a banach algebra A with identity e is defined to be $\{\phi \in A^*: \phi(e) = \|\phi\| = 1\}$. An element $h \in A$ is hermitian if $\|e^{i\lambda h}\| = 1$ for all real λ. Equivalently [2] h is hermitian if and only if $\{\phi(h): h \in S\} \subseteqq \boldsymbol{R}$. A^{**} when endowed with Arens multiplication [3] is a Banach algebra with identity e, and by the weak-star density of A in A^{**}, $h \in A^{**}$ is hermitian if and only if h is real valued on the state space of A.

In [5] it is shown that M-ideals in Banach algebras are necessarily subalgebras. Other results of this paper and [6] needed in the sequel are now summarized:

Let M be an M-ideal in $B(\ell_p)$, $1 < p < \infty$. Then clearly $M^{\perp\perp}$ is an M-summand in $B(\ell_p)^{**}$; that is, $B(\ell_p)^{**} = M^{\perp\perp} \oplus_{c_0} M^\#$. Let