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# ABSOLUTE $C^*$-EMBEDDING OF $F$-SPACES

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**Let $\mathscr{U}$ be an open cover of a space $X$. We define $\mathscr{U}$ to be a $P$-cover if each element of $\mathscr{U}$ is a proper subset of $X$, $\mathscr{U}$ is closed under countable unions and for every $U \in \mathscr{U}$ there is a $V \in \mathscr{U}$ such that $U$ and $X \backslash V$ are completely separated. We prove an $F$-space $X$ is $C^*$-embedded in every $F$-space it is embedded in iff $X$ has no $P$-covers or $X$ is almost compact.**

1. Introduction. In 1949, Hewitt [7] proved that a Tychonoff space is $C^*$-embedded in every Tychonoff space in which it is embedded iff $X$ is almost compact. C. E. Aull [1] has shown that a $P$-space $X$ is $C^*$-embedded in every $P$-space in which it is embedded iff $X$ is almost Lindelöf (given disjoint zero sets of $X$ at least one is Lindelöf). These two theorems are examples of absolute $C^*$-embedding theorems. In §3 of this paper we will provide the absolute $C^*$-embedding theorem for $F$-spaces. In §4 we obtain partial results concerning $C^*$-embeddings in basically disconnected spaces.

2. DEFINITIONS. All topological spaces will be assumed to be Tychonoff. The following theorem is useful when dealing with $F$-spaces and also provides a definition of $F$-spaces.

THEOREM 2.1 [6, 14.25]. *The following are equivalent*
(1) *$X$ is an $F$-space.*
(2) *$\beta X$ is an $F$-space.*
(3) *disjoint cozero subsets of $X$ are completely separated.*
(4) *cozero subsets of $X$ are $C^*$-embedded.*
(5) *disjoint cozero subsets of $\beta X$ have disjoint closures.*

$X$ is *basically disconnected* if the closure of every cozero set is clopen. $X$ is a *$P$-space* if every zero set of $X$ is open. The reader is referred to [6] for background on $P$-spaces, $F$-spaces and basically disconnected spaces. $X$ is *weakly Lindelöf* if every open cover of $X$ contains a countable subcollection whose union is dense in $X$ [2]. If $X$ is a subspace of $Y$ and $\mathscr{C}$ is a collection of subsets of $Y$, we define $\mathscr{C}|_X = \{C \cap X: C \in \mathscr{C}\}$.

The cardinality of a set $K$ is denoted by $|K|$ and the immediate successor of a cardinal $\alpha$ is denoted by $\alpha^+$. The cofinality of a non-