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A MORITA CONTEXT RELATED TO FINITE AUTOMORPHISM GROUPS OF RINGS

MIRIAM COHEN

Let R be a semiprime ring, G a finite group of automorphisms of R and R^G the fixed ring. We investigate the associated Morita context (R^G, R, R, R^*G), where R^*G is the skew group ring. We then apply these results to two situations: (1)G is X-outer (2)R is $|G|$-torsion free.

0. Introduction and preliminaries. Let R be a ring, G a finite group of automorphisms of R and $R^G = \{x \in R \mid x^g = x$ for all $g \in G\}$. There has been considerable interest in the past years in studying connections between R^G and R. The two major ways to approach the subject were the direct approach, and via the skew group ring $R*G$ which we denote by S. In this paper we investigate a third way which was used in the commutative case by Chase, Harrison and Rosenberg [5], and was suggested to us by S.A. Amitsur.

We consider an associated Morita context $[R^G, R, R, S]$ with $(x, y) = \sum_{g \in G} (xy)^g$ and $[x, y] = \sum_{g \in G} xy^{g^{-1}}g$, for all $x, y \in R$. This context incorporates all the relevant ingredients. The fixed ring is known to be nonzero in three major situations: (1) [11] R is semiprime and G is X-outer. (2) [4] R is semiprime and $|G|$-torsion free (3) [11] R has no nilpotent elements. Since in the third situation $t_G(x) = \sum_{g \in G} x^g$ might turn out to be 0 for all $x \in R$ [7], we apply the results of § 1 only to the first two cases. It seems however plausible that by changing the context one could deal with the third situation by similar techniques.

Throughout the paper we assume that R is a semiprime ring, an immediate consequence of which is that [,] is a nondegenerate bilinear form. Another consequence is: if $\mathscr{N}(S) = 0$ where $\mathscr{N}(*)$ is the prime, Jacobson, locally nilpotent or nil radical, then $\mathscr{N}(R^G) = 0$ [Lemma 1.2]. In § 1B we investigate properties of the context when also (,) is assumed to be nondegenerate. We prove, among the rest, that (R, R) is essential in R^G and that when R^G is semiprime then R is Goldie (Artinian) iff R^G is Goldie (Artinian); when R is Artinian then (,) is onto. [Theorem 1.6 and Lemma 1.3.] Some of the results were proved by Montgomery [15]. Since R is semiprime, it is a faithful R^G-module, however, it need not be a faithful S-module. In § 1C we investigate the annihilator of R in S, which turns out to be the annihilator (right or left) in S of a two sided ideal of S, namely the ideal $[R, R]$. When (,) is non-