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A NOTE ON LINEARLY ORDERED NET SPACES

JAMES R. BOONE

The linearly ordered net spaces are introduced in this paper. This concept is a generalization of both the sequential spaces and the linearly ordered base spaces. The fundamental properties of this class, including mapping properties, are presented. Various applications of a general nature are given as well as some applications to weak covering axioms. The class of lo-net spaces is characterized as the class of well-ordered net spaces.

I. **Introduction.** In this note the large class of linearly ordered net (lo-net) spaces is introduced, the fundamental properties of this class are presented and the study of applications is initiated. The lo-net spaces are very useful simultaneous generalizations of sequential spaces and linearly ordered base spaces. The applications of sequential spaces are numerous and well known. The linearly ordered base (lob) spaces were studied by Davis [6] and applications of a general nature were presented along with the beginnings of important applications to weak covering axioms. In [7] the applications of lob-spaces to various weak covering axioms were studied extensively.

Section 2 contains the definitions of the notions associated with the linearly ordered net spaces, as spaces with the weak topology generated by a class of subsets. The fundamental structural properties and some of the basic applications are presented in §3. In §4 some of the results of [6] and [7] are reexamined from the broadened view of lo-net spaces. The subtle differences between lo-net and very lo-net spaces are illustrated by examples. In particular, Lemma 2.3.1 of [7] is not true and a corrected version is presented along with its lo-net version. Finally the true nature of the lo-net spaces as a generalization of sequential spaces and lob-spaces is revealed in §5, where the class of lo-net spaces is characterized as the class of well-ordered net spaces.

II. **Preliminaries.** A linearly ordered net (lo-net) is a net whose directed set is linearly ordered. The collection of linearly ordered nets $\mathscr{L}_X$ in a space X determines a natural cover [9] of X. Many of the notions in this study are specific applications of concepts and properties introduced and developed by Stan Franklin [9]. A topological space X will be called a *linearly ordered net* (*lo-net*) *space* provided $H \subset X$ is closed if and only if for every convergent lo-net in H, say $x_\lambda \to x$, we have $x \in H$. That is X is a lo-not space