FACIFIC JOURNAL OF MATHEMATICS
Vol. 98, No. 1, 1982

CARTAN SUBALGEBRAS OF BANACH-LIE ALGEBRAS OF OPERATORS

HUMBERTO R. ALAGIA

A method to determine the Cartan subalgebras of certain real Banach-Lie algebras of operators is described. The algebras in question are the real, simple, separable, infinite dimensional L*-algebras, realized as algebras of Hilbert-Schmidt operators.

1. Introduction. Banach-Lie algebras appear in many different contexts. One particularly interesting class of examples is provided by L^*-algebras, first studied by Schue ([7], [8]). An L^*-algebra L is a Lie algebra (over the real or complex field) whose underlying vector space is a Hilbert space together with a "star map" $x \mapsto x^*$ satisfying the condition $\langle [x, y], z\rangle = \langle y, [x^*, z]\rangle$ for every x, $y, z \in L$. Here $\langle , \rangle$ denotes the inner product in the Hilbert space L and $[\,,]$ the product in the Lie algebra L.

If L is a finite dimensional semisimple Lie algebra over the complex field, then every compact real form L_0 of L determines a structure of L^*-algebra. Indeed, if σ denotes the conjugation of L with respect to L_0 define an inner product by $\langle x, y\rangle = B(x, \sigma(y))$ for every $x, y \in L$, where B is the Killing form of L, and set $x^* = -\sigma(x)$. Then it is easily checked that $(L, \langle , \rangle, {}^*)$ is a complex L^*-algebra.

Infinite dimensional examples are provided by the associative H^*-algebras of Ambrose [1], considered as Lie algebras by setting $[x, y] = xy - yx$.

An L^*-algebra is semisimple if its center is $\{0\}$. This paper is concerned with infinite dimensional, separable L^*-algebras which are moreover simple (i.e., do not contain any closed ideals different from $\{0\}$ and L). For all definitions and basic facts of the general theory, see [7], [8], [5].

Let $\mathscr{H}$ denote an infinite dimensional, separable complex Hilbert space. The set $L_2(\mathscr{H})$ of all Hilbert-Schmidt operators on $\mathscr{H}$ has a structure of L^*-algebra, where $\langle , \rangle$ is the usual inner product defined on $L_2(\mathscr{H})$ and the $*$-operation is the operation of taking the adjoint of an operator T. This L^*-algebra will be denoted by L_A; it is a simple L^*-algebra of type A and all other simple L^*-algebras are isomorphic (up to a multiple of the inner product) to a (real or complex) L^*-subalgebra of L_A. In particular L_B and L_C will denote the simple, complex L^*-algebras of type B and C respectively, considered as subalgebras of L_A. For details see [7], [3], [4], [10].

In the following list $\tilde{L}$ denotes a simple complex L^*-algebra (L_A,