PACIFIC JOURNAL OF MATHEMATICS
Vol. 98, No. 2, 1982

DIAGONALIZATION UP TO WITT

MAX WARSHAUER

In this paper we construct an additive system of generators for the Witt group of u-Hermitian inner product spaces.

The idea to do this was suggested by Conner, who should be thanked for his help on this and other matters. The reader should note that the additive system of generators obtained is for $H_u(K)$, where K is a fractional ideal in an algebraic number field. In this setting there are in general no rank one forms. This contrasts sharply with the case of $H_1(O(K))$ for the Dedekind ring of integers $O(K)$ in an algebraic number field which has been studied and understood [1].

The case of a fractional ideal arises in [3], for example, where the group $H_{\pm 1}(\mathscr{D}^{-1}(E/Q))$ for $K = \mathscr{D}^{-1}(E/Q)$ the inverse different is computed. However, no method for constructing representatives of the Witt classes is discussed in [3].

In [2], the group $H_{-1}(Z(\lambda))$, where $\lambda = \exp 2\pi i/p$ a pth root of unity, p an odd prime, was studied. Here rank 2 forms were constructed by *ad hoc* considerations. We return to this example later, as it is a special case of the general result we obtain.

We now begin by describing the setting in which we work. Let E be an algebraic number field together with an involution —. The fixed field of — is F. We denote by $O(E)$ and $O(F)$ the Dedekind rings of integers in E and F respectively. Let K be a — invariant fractional $O(E)$-ideal. We fix u a unit in E of norm 1, i.e., $u\bar{u} = 1$.

DEFINITION 1. A K-valued inner product space is a pair (M, B) satisfying:

(1) M is a finitely generated torsion free $O(E)$-module.

(2) B: $M \times M \to K$ is a non-singular K-valued u-Hermitian inner product defined on M.

The non-singularity condition is that the adjoint map $Ad_R B$: $M \to \mathrm{Hom}_{O(E)}(M, K)$ defined by $m \to B(-, m)$ is an isomorphism. Further, B is u-Hermitian, meaning

(a) $B(x, y) = u\overline{B(y, x)}$ for x, y in M.

(b) $B(\lambda x, y) = B(x, \bar{\lambda} y) = \lambda B(x, y)$ for λ in $O(E)$.

Note that when $u = +1$, this is the usual notion of Hermitian. When $u = -1$, we have skew-Hermitian. The generalization to u-