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THE KREIN-MILMAN PROPERTY AND COMPLEMENTED BUSHES IN BANACH SPACES

AGGIE HO

We give "complementation" as a sufficient condition on a bush in a Banach space for the space to fail the Krein-Milman property. We also construct an example of a Banach space X which contains a complemented bush. Hence the space X fails the Krein-Milman property. However the closed convex span of the bush contains infinitely many extreme points and no denting points. Moreover, the closed convex span of these extreme points contains the original bush.

1. Introduction. It is an open question whether a *nondual* Banach space with the Krein-Milman property has the Radon-Nikodým property. Since a Banach space contains a bush if and only if it does not have the Radon-Nikodým property, a space which contains a bush and satisfies the Krein-Milman property would settle this question. Theorem A indicates that such a space must not have a complemented bush. For a summary of results on the Radon-Nikodým property and the Krein-Milman property, see [**1**, Ch. VII].

A Banach space X is said to have the *Krein-Milman property* if every bounded closed convex subset in X is the closed convex span of its extreme points. We define a *bush* in a Banach space X to be a subset

$$B = \{x^{ni}: 1 \leqq i \leqq N(n),\ n \geqq 1\}$$

of the unit ball of X that satisfies the following conditions:

(B1) For each $n \geqq 1$, the collection of the first $N(n+1)$ positive integers is the union of $N(n)$ consecutive sets

$$\{S_i^{n+1}: 1 \leqq i \leqq N(n)\}$$

such that each S_i^{n+1} has $r_i^{n+1} \geqq 2$ members (the bush is a *tree* if each $r_i^n = 2$) and

$$x^{ni} = \frac{1}{r_i^{n+1}} \sum \{x_j^{n+1}: j \in S_i^{n+1}\} .$$

(B2) There is a positive *separation constant* ε such that, for each i and every $j \in S_i^{n+1}$, the following holds:

$$\|x^{ni} - x^{n+1,j}\| > \varepsilon .$$

We say $x^{n+1,j}$ follows x^{ni} or $(n+1, j) > (n, i)$ if $j \in S_i^{n+1}$. The