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RITT SCHEMES AND TORSION THEORY

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There is a natural way to associate a torsion theory to any differential ring. Using this tool, one may prove that there is a duality between the category of reduced affine Ritt schemes and a full subcategory of the category of Ritt algebras. As a consequence, a brief investigation is made concerning morphisms of differential finite type and a differential version of Chevalley's constructibility theorem is proved for such morphisms.

1. Introduction. The category *Diff* has as its objects commutative rings A with unit together with m derivation operators $D_1, \cdots, D_m: A \to A$ which commute. A morphism $f: (A, D_1, \cdots, D_m) \to (\bar{A}, \bar{D}_1, \cdots, \bar{D}_m)$ in *Diff* is a ring homomorphism $f: A \to A$ with $\bar{D}_i f = f D_i$ for every $i = 1, \cdots, m$. Recall from [5, p. 110] that an LDR-space is pair $(X, \mathcal{O}_X)$ where X is a topological space and $\mathcal{O}_X$ is a sheaf in *Diff* on X such that for each $P \in X$ the ring $\mathcal{O}_{X,P}$ is local and its maximal ideal is differential. A morphism of LDR-spaces $f: (X, \mathcal{O}_X) \to (Y, \mathcal{O}_Y)$ is a pair (ψ, θ) where $\psi: X \to Y$ is continuous and $\theta: \mathcal{O}_Y \to \psi_* \mathcal{O}_X$ is a morphism of sheaves in *Diff* on Y such that for each $P \in X$, the morphism $\mathcal{O}_{Y,\psi(P)} \to \mathcal{O}_{X,P}$ is local. The category of LDR-spaces is denoted by LDR.

There exist two fundamental functors $\mathrm{Spec}_D: Diff \to LDR$ and $\Gamma_D: LDR \to Diff$ defined such as follows: for any differential ring A, the topological space $\mathrm{Spec}_D A$ consists of the prime differential ideals of A, the topology being induced by the natural inclusion $j: \mathrm{Spec}_D A \to \mathrm{Spec}\, A$ and the defining sheaf $\mathcal{O}_{\mathrm{Spec}_D A}$ being $\hat{A} = j^{-1}(\tilde{A})$, where $\tilde{A}$ is the defining sheaf of the scheme $\mathrm{Spec}\, A$, [3, p. 70]. Note that $\tilde{A}$ (and consequently $\hat{A}$) has a natural structure of sheaf in *Diff*. Indeed the derivations $D_1, \cdots, D_m$ on A canonically give derivations $D_{1,P}, \cdots, D_{m,P}$ on A_P for any $P \in \mathrm{Spec}\, A$; hence for any open set $U \subseteq \mathrm{Spec}\, A$, the ring $\Gamma(U, \tilde{A})$ becomes a differential ring with derivations $D_{1,U}, \cdots, D_{m,U}$ defined such as follows: for any $s \in \Gamma(U, A)$, $D_{i,U}(s)$ is the section defined by the family $\{D_{i,P}(s_P)\}_{P \in U}$. On the other hand for any LDR-space X, $\Gamma_D(X)$ will denote the differential ring of global sections $\Gamma(X, \mathcal{O}_X)$.

It was proved in [5] that at least in the case of a single derivation, the functors Spec_D and Γ_D give an adjunction between *Diff* and LDR^{op}. We shall prove in §2 that these functors give in fact an equivalence between sufficiently large subcategories.

For the remainder of this paper we shall suppose that all rings