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THE CLASSIFICATION OF UNIFORM ALGEBRAS ON PLANE DOMAINS

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Let Ω be an open subset of the complex plane. Denote by $\mathcal{O}(\Omega)$ the algebra of all holomorphic functions on Ω, equipped with the topology of uniform convergence on compact set. The object of this paper is to provide a complete classification of all the closed subalgebras of $\mathcal{O}(\Omega)$ which contain the polynomials, and apply this classification to several concrete problems, including localness of these algebras, continuity of homomorphisms, and number of generators. It should be emphasized that no assumptions are made as to the connectivity of Ω. In fact, in the cases of most interest, Ω will not be connected and some of the connected components of Ω will not be simply connected.

The most natural way in which such a classification might proceed would be to show, for such an algebra A, that the space ΔA of nonzero, continuous complex-valued homomorphisms of A, can be equipped with the structure of a one-dimensional complex-analytic space. Unfortunately, this is not generally the case. However, we can realize ΔA as the quotient of a certain Riemann surface, and equip it with a "pseudo-analytic structure", and this data serves to classify the algebra A. This is accomplished in §§ 1 and 2.

As with any classification scheme, it is natural to ask whether the scheme described here is a satisfactory one. This is partly a question of taste, but we suggest that a reasonable criterion is applicability to the solution of concrete problems. Following a suggestion of Kaplansky [11, p. 12] we consider three "test problem" which arise naturally in the study of uniform algebras:

(1) Is the algebra local on its homomorphism space?

(2) Is every complex-valued homomorphism of the algebra necessarily continuous?

(3) Is the algebra finitely-generated?

In § 3, we use our classification scheme to show that the first two test problems always have affirmative solutions, and that the third test problem has an affirmative solution if Ω has only a finite number of connected components. We also give another application, suggested by work of Gamelin [8] and Bjork and de Paepe [7].

Some of our methods work in more general contexts, although the sharpest results do not obtain. In § 4, we indicate the sort of result which can be obtained, and some limitations.