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CHARACTERIZATION AND ORDER PROPERTIES OF PSEUDO-INTEGRAL OPERATORS

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Let $(X, \mathscr{A}, m_1)$ and $(Y, \mathscr{B}, m_2)$ be separable σ-finite measure spaces. A linear transformation T from an order-ideal L of measurable functions on Y into the space of measurable functions on X is called a pseudo-integral operator if it is induced by a measure μ on $X \times Y$ via the equation

$$\int (Tf)(x)g(x)m_1(dx) = \iint f(y)g(x)\mu(dx, dy)$$

for sufficiently many functions g. Our main theorem states that T is a pseudo-integral operator if and only if $Tf_n \to 0$ a.e. whenever $0 \leqq f_n \leqq f \in L$ and $f_n \to 0$ a.e. We also study the order structure of the class of pseudo-integral operators showing that they form a band (order-closed ideal) in the space of order-bounded operators.

Introduction. Let $(X, \mathscr{A}, m_1)$ and $(Y, \mathscr{B}, m_2)$ be separable σ-finite measure spaces, and let $M(X)$ and $M(Y)$ be the linear spaces of equivalence classes of (real or complex) measurable functions on X and Y respectively. A linear operator T from a linear subspace L of $M(Y)$ into $M(X)$ is called an integral operator if there exists a measurable function k on $X \times Y$ (called the kernel of T) such that for every f in L, Tf is given by the equation $(Tf)(x) = \int f(y)k(x, y)m_2(dy)$ for m_1—almost every x. Arveson [2] introduced a more general class of operators, which he called pseudo-integral operators, associated with measures, rather than functions, on $X \times Y$. By a pseudo-integral operator we mean an operator given by the equation

$$\int (Tf)(x)g(x)m_1(dx) = \iint f(y)g(x)\mu(dx, dy) , \tag{0.1}$$

for sufficiently many functions g (this will be made precise later). If $(Y, \mathscr{B})$ is a standard Borel space, then T can be given explicitely by the equation

$$(Tf)(x) = \int f(y)\mu_x(dy) \tag{0.2}$$

where $\{\mu_x\}$ is a certain family of measures on Y, related to μ via the theorem on disintegration of measures.

The purpose of this paper is to give a necessary and sufficient condition for an operator between spaces of measurable functions to