PACIFIC JOURNAL OF MATHEMATICS
Vol. 99, No. 1, 1982

DECOMPOSITIONS OF HOMOGENEOUS CONTINUA

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F. Burton Jones was the first to study properties of homogeneous continua by means of decompositions. In 1951, he showed [12] that a hereditarily unicoherent, homogeneous curve must be indecomposable. His proof used an aposyndetic decomposition.

Soon after, he [11] extended the result to the Jones Aposyndetic Decomposition Theorem. For the case of a decomposable planar continuum M, the theorem states that M admits a continuous decomposition such that the decomposition space is a simple closed curve and each element of the decomposition is an indecomposable, homogeneous, acyclic continuum. Recently, the author [13] has improved Jones' theorem in the nonplanar case (the improved version is stated in this paper as Theorem 2).

R. H. Bing [1] proved that the pseudo-arc is homogeneous. Because of Jones' theorem, it would have been natural to ask if there were a planar continuum that admitted a decomposition into pseudo-arcs with the quotient space being a simple closed curve. Such an example was obtained independently by Bing and by Jones in 1954 [3].

C. L. Hagopian and the author [9] have proved that any nonplanar, homogeneous, circle-like continuum that is not a solenoid admits a continuous decomposition into pseudo-arcs such that the quotient space is a solenoid. The author [14] has constructed such a continuum for each solenoid.

Jones [10] and Hagopian [6 and 7] have used decompositions of a proper subcontinuum to investigate properties of homogeneous, plane continua.

David Bellamy and Lewis Lum have communicated to the author that they will use decompositions to prove that a uniquely-arcwise connected continuum cannot be homogeneous.

Clearly, decompositions are a powerful and frequently-used tool in investigating homogeneous continua.

The purpose of this paper is to present a general theory of decomposition of homogeneous spaces. All the decompositions in the theorems above fit into this theory. Theorem 1 presents a special (absolute) case of the theory, easy to state and with enough strength to imply most of the decompositions for curves. We derive through