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ON PRODUCT BASES

JOHN C. MORGAN II

Starting with the notion of a category base, which is a generalization of a topological space, we investigate cartesian products of category bases. Extension are obtained of several classical topological theorems concerning Baire category in product spaces.

In §1 we formulate the basic concepts and state relevant theorems which were established earlier. In §2 we give conditions under which the cartesian product of two category bases is also a category base. General properties of product bases are then presented in §3. Utilizing the notion of a separable category base, discussed in §4, we establish generalizations of results of Kuratowski and Ulam, Sikorski, and Oxtoby on cartesian products (cf. [2], [10], [11]). Even in the context of topological spaces the theorems given are more general than the previous results.

1. **Preliminaries.** In this section we review the basic concepts and result which are pertinent to this article. We have also added several examples to clarify the ideas. For proofs of the main theorems see [4], [5], [8]. For other related results see [6], [7], [9].

DEFINITION. A pair $(X, \mathscr{C})$, where $\mathscr{C}$ is a family of subsets of a nonempty set X, is called a category base if the nonempty sets in $\mathscr{C}$, called regions, satisfy the following axioms:

1. Every point of X belongs to some region; i.e., $X = \bigcup \mathscr{C}$.
2. Let A be a region and let $\mathscr{D}$ be any nonempty family of disjoint regions which has power less than the power of $\mathscr{C}$.

(a) If $A \cap (\bigcup \mathscr{D})$ contains a region then there is a region $D \in \mathscr{D}$ such that $A \cap D$ contains a region.

(b) If $A \cap (\bigcup \mathscr{D})$ contains no region then there is a region $B \subset A$ which is disjoint from every region in $\mathscr{D}$.

Among the examples of category bases are the following.

EXAMPLE 1A. The family of all complements of finite subsets of an uncountable set.

EXAMPLE 1B. Every topology.

EXAMPLE 1C. The family of all measurable sets of positive