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PROJECTIVE SPACE AS A BRANCHED COVERING WITH ORIENTABLE BRANCH SET

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We show that if RP^n is a smooth k-fold branched covering of S^n with orientable branch set and either $k \leqq n$ or the singular set of the covering is connected, then $n = 1, 3$, or 7.

1. Introduction. Let $\tilde{M}$ and M be smooth, closed n-manifolds. A k-fold branched covering is a smooth map $f: \tilde{M} \to M$ together with smoothly imbedded, codimension 2 submanifolds $K \subset M$ and $\tilde{K} = f^{-1}(K) \subset \tilde{M}$, such that f restricted to $\tilde{M} - \tilde{K}$ is a k-fold covering, f restricted to $\tilde{K}$ is a finite covering onto each component of K, and the degree of f is k. Brand [2] suggests the problem of determining the values of n for which RP^n, real projective n-space, is a branched covering of S^n and he shows that if RP^n is a branched covering of any manifold with trivial Stiefel-Whitney class, such as S^n, then $n = 2^t \pm 1$. We show that the possible values of n can be further limited if the branch set of the covering, K, is orientable and either $k \leqq n$ or the covering is simple, that is, the singular set, the subset of $\tilde{K}$ where f fails to be a local diffeomorphism, is connected.

THEOREM 1.1. *If RP^n is a k-fold branched covering of a π-manifold with orientable branch set and either $k \leqq n$ or the covering is simple, then $n = 1, 3$, or 7.*

A π-manifold is a smooth, closed n-manifold with stably trivial tangent boundle, such as S^n. Our definition of simple branched covering is due to Hilden, [5]. Brand and Brumfiel have extensively studied simple branched coverings having the special property that the connected singular set is all of $\tilde{K}$, [3].

The identity map provides a branched covering of S^1 by RP^1 and it is well known that RP^3 is a 2-fold branched covering of S^3. Hilden and Montesinos have shown, independently, that every closed orientable 3-manifold is a simple, 3-fold branched covering of S^3, [5], [7].

2. Normalized branched coverings. In [2], Brand defines a normalized branched covering. He uses this concept to show that there is a certain K-theoretic necessary condition for the existence of a branched covering.