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THE WEAK NULLSTELLENSATZ FOR FINITE DIMENSIONAL COMPLEX SPACES

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Two of the most important global properties of complex spaces $(X, \mathscr{O})$, holomorphic convexity and holomorphic separability, can each be characterized in terms of the standard natural map $\chi: X \to S_c(\mathscr{O}(X))$, $x \to \chi_x$, $\chi_x(f) := f(x)$, $f \in \mathscr{O}(X)$, of X into the continuous spectrum $S_c(\mathscr{O}(X))$ of the global function algebra $\mathscr{O}(X)$. The question as to whether there is any global function theoretical property of $(X, \mathscr{O})$ corresponding to the surjectivity of χ has remained unanswered. The purpose of this paper is to present an answer for finite dimensional spaces. For such spaces $(X, \mathscr{O})$ it will be shown that the surjectivity of χ is equivalent to requiring that for finitely many functions $f_1, \cdots, f_m \in \mathscr{O}(X)$ with no common zero on X there exist functions $g_1, \cdots, g_m \in \mathscr{O}(X)$ with $\sum_{i=1}^m f_i g_i = 1$. This property will be called the weak Nullstellensatz for the complex space $(X, \mathscr{O})$. An example due to H. Rossi shows that this result is not valid for infinite dimensional complex spaces. An application of the weak Nullstellensatz for Fréchet algebras A involving the Michael conjecture is that $S_c(A)$ is always dense in the spectrum $S(A)$ of A.

1. Introduction. Important global properties of complex spaces[1) $(X, \mathscr{O})$ can be characterized in terms of the standard natural map $\chi: X \to S_c(\mathscr{O}(X))$, $x \to \chi_x$, $\chi_x(f) := f(x)$, $f \in \mathscr{O}(X)$, of X into the continuous spectrum $S_c(A)$ of the global function algebra $A := \mathscr{O}(X)$ which takes points x of X to the corresponding point evaluations χ_x. For example: X is holomorphically separable if and only if χ is injective; X is holomorphically convex if and only if χ is proper (see § 3). A complex space which is both holomorphically separable and holomorphically convex is called Stein. The customary description of Stein spaces as being those complex spaces which have "sufficiently many" global holomorphic functions [14, VII] attains precision from a theorem of Igusa/Remmert/Iwahashi/Forster [15, 21, 17, 7] stating: X is Stein if and only if χ is a homeomorphism. In other words, a complex space X is Stein if and only if there are enough global holomorphic functions on X to enable X to be regained topologically from the continuous spectrum of these functions.[2) According to the above remarks, the main assertion of this theorem is that

[1) Throughout this paper, a complex space means a reduced complex space with countable topology.

[2) A Stein space X can also be regained as a complex space from $S_c(A)$ [**8**, **13**].