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RADICAL CLASSES OF REGULAR RINGS WITH ARTINIAN PRIMITIVE IMAGES

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This paper deals with radical classes consisting of regular rings, all of whose primitive homomorphic images are artinian (such rings will be called PA-regular). Noteworthy examples of such radical classes include, for each n, the class of regular rings satisfying the condition

$$a \text{ nilpotent} \Longrightarrow a^n = 0 ,$$

and thus, in particular, the class of all strongly regular rings. It is shown that every radical class $\mathscr{R}$ consisting of PA-regular rings is hereditary, and is the lower radical class defined by those of its members which are isomorphic to matrix rings of strongly regular rings with identities.

Moreover, for each m, the class of strongly regular rings for which the $m \times m$ matrix ring is in $\mathscr{R}$ is a radical class. This fact establishes a very important rôle for the radical classes of strongly regular rings, and accordingly a section is devoted to the latter. In the final section some attention is given to the question of closure of radical classes under direct products.

It is probably safe to say that more is known about the radical classes which contain all nilpotent rings than about those which contain none. Certainly—with the (important) exception of the semisimple radical classes—the radical classes which have been most studied are towards the supernilpotent end of the spectrum.

The present paper has three main aims:

(1) to present some examples of (hereditary) subidempotent radical classes;

(2) to examine radical theory in a relatively tractable but nontrivial class of rings—the regular rings whose primitive homomorphic images are all artinian;

(3) to say something about radical classes which are closed under directs.

Investigations akin to objective (2), wherein radical theory is studied "in microcosm", may be regarded as compromise substitutes for the (probably unrealistic) aim of describing all radical classes.

The question of dirct product closure has been around for a long time. It is easy to see that a hereditary radical class with this closure property must be either supernilpotent of subidempotent, and we shall here be concerned with the latter, providing a few examples and counterexamples and answering, in the negative, Richard Wiegandt's