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NOTE ON THE SPACES $P(S)$ OF REGULAR PROBABILITY MEASURES WHOSE TOPOLOGY IS DETERMINED BY COUNTABLE SUBSETS

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Two closely connected topics are discussed: countable tightness in the spaces $P(S)$ of regular probability measures with the weak topology and a convex analogue to Lindelöf property of the weak topology of the function spaces $C(S)$ defined by H. H. Corson. The main result of this note exhibits a rather wide class of compact spaces stable under standard operations including the operation $P(S)$, such that within this class both of the properties we deal with are dual each other and they behave in a regular way. Some related open problems are stated.

1. Introduction. In this note we consider two closely connected topics: countable tightness in the spaces $P(S)$ of regular probability measures on compact spaces endowed with the weak* topology and property (C)—a convex analogue to Lindelöf property of the weak topology of function spaces $C(S)$ defined by H. H. Corson [6] (for the terminology and definitions see §§ 2 and 3).

Our results are related to the following two problems:

(A) *Property* (C) *of* $C(S)$ *is equivalent to a property of* $P(S)$ *which is a convex analogue to countable tightness* (*see Lemma* 3.2). *This property is a priori weaker than countable tightness but no example known to us shows that this is realy the case. So, for what compact spaces* S *countable tightness of* $P(S)$ *is equivalent to property* (C) *of* $C(S)$, *or putting this another way, when property* (C) *and countable tightness are dual each other?*

(B) *Does the function space* $C(S \times S)$ *or* $C(P(S))$ *have property* (C) *provided that the space* $C(S)$ *has this property? Does countable tightness of the space* $P(S \times S)$ *or* $P(P(S))$ *follow from countable tightness of the space* $P(S)$?

It should be mentioned here that the only examples we know of compact spaces S with countable tightness for which $C(S)$ fails to have property (C) or $P(S)$ fails to have countable tightness, due to Haydon [14] and to van Douwen and Fleissner [7], are constructed under additional set theoretic hypotheses. This yields yet another problem, whether in such examples some extra axioms for set theory are necessary (the results of this note, however, have no connection to this question).