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ON THE REPRESENTATION THEORY OF RINGS IN MATRIX FORM

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We study the category of modules for rings of the form

$$\begin{pmatrix} \Lambda_1 & N \\ M & \Lambda_2 \end{pmatrix}$$

where M is a $\Lambda_2 - \Lambda_1$-bimodule and N is a $\Lambda_1 - \Lambda_2$-bimodule. We first obtain a structural result and then study special cases of such rings. The goal is to reduce the study of modules over such rings to modules over generalized lower triangular rings.

Rings of the form

$$\Gamma = \begin{pmatrix} \Lambda_1 & 0 \\ M & \Lambda_2 \end{pmatrix} \tag{0.1}$$

where Λ_1, Λ_2 are rings and M is a $\Lambda_2 - \Lambda_1$-bimodule have appeared often in the study of the representation theory of Artin rings and algebras. We list just a few references ([2, §4], [3], [4], [5, §9 and appendix], [6], [7, §2], [9] and [10]). Such rings appear naturally in the study of homomorphic images of hereditary Artin algebras. For, if Ω is such an algebra, since Ω must have a simple injective left module it follows that Ω can be put in the form (0.1) with Λ_1 a semisimple Artin ring. One of the main reasons rings of the form (0.1) are so useful is that their modules can be studied by knowing the Λ_1 and Λ_2 modules together with certain homomorphisms. In particular, we have

(0.2) THEOREM. [4] *Let Γ be the ring*

$$\begin{pmatrix} \Lambda_1 & 0 \\ M & \Lambda_2 \end{pmatrix}.$$

The category of left Γ-modules is equivalent to the following category: the objects are triples (X, Y, f) where X is a left Λ_1-module, Y is a left Λ_2-module and $f \in \mathrm{Hom}_{\Lambda_2}(M \otimes_{\Lambda_1} X, Y)$. The morphisms $\alpha\colon (X, Y, f) \to (X', Y', f')$ are pairs $\alpha = (\alpha_1, \alpha_2)$ where $\alpha_1 \in \mathrm{Hom}_{\Lambda_1}(X, X')$, $\alpha_2 \in \mathrm{Hom}_{\Lambda_2}(Y, Y')$ such that the following diagram commutes