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REAL HOMOLOGY OF LIE GROUP HOMOMORPHISMS

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Let $h: G_1 \to G_2$ be a homomorphism of compact, connected Lie groups and let $h_*: H_*(G_1) \to H_*(G_2)$ be the homomorphism of homology with real coefficients induced by h. The investigation of the properties of h that can be deduced from a knowledge of h_* goes back at least to work of Dynkin in the early 1950's. This paper presents several contributions to the investigation. The main result is a characterization of homomorphisms with abelian images as those whose induced homomorphisms annihilate all three-dimensional indecomposables. We then examine what the homology can tell us about the dimension of the abelian image. Next, an inequality relating the homology of the kernel of h to the kernel of h_* leads to sufficient conditions for h to have an abelian, semisimple, or finite kernel. The final sections present various relationships between h_* and the kernel and image of h and, in particular, show that if $h(G_1)$ is totally nonhomologous to zero in G_2, then h_* gives quite precise information about the behavior of h.

For the results of Dynkin, see [2] and [3].

In order to avoid frequently repeating the same hypotheses, we state that throughout this paper:

HYPOTHESES. *h: $G_1 \to G_2$ is a homomorphism where G_1 and G_2 are compact, connected Lie groups.*

1. Homomorphisms with abelian images.

A homomorphism $h: G_1 \to G_2$ induces homomorphisms of real homology $h_{*s}: H_s(G_1) \to H_s(G_2)$ for all s. We will need the following observation:

LEMMA 1.1. *If $h: G_1 \to G_2$ is surjective, then $h_{*s}: H_s(G_1) \to H_s(G_2)$ is surjective for all s.*

Proof. Since h is surjective, there is an isomorphism $\bar{h}: G_1/K \to G_2$, where K is the kernel of h. The quotient homomorphism $q: G_1 \to G_1/K$ induces a surjection of real homology, so the commutativity of