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COMPACTNESS PROPERTIES OF ABSTRACT KERNEL OPERATORS

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Let E and F be two Banach function spaces on two σ-finite measure spaces (Y, Σ, ν) and (X, S, μ) respectively. Then an operator $T: E \to F$ is called a kernel operator, if there exists a $\mu \times \nu$-measurable real valued function $K(x, y)$ on $X \times Y$ such that for each $f \in E$ we have

$$Tf(x) = \int_Y K(x, y) f(y) d\nu(y) \text{ for } \mu\text{-almost all } x, \text{ and}$$

$$\int_Y |K(\cdot, y) f(y)| d\nu(y) \in F .$$

It is well known that every kernel operator belongs to the band $(E' \otimes F)^{dd}$ generated by the finite rank operators, and under certain conditions $(E' \otimes F)^{dd}$ consists precisely of all kernel operators.

In this paper we consider E and F to be two locally convex-solid Riesz spaces. Motivated by the above remarks, we call every operator in $(E' \otimes F)^{dd}$ that can be written as a difference of two positive weakly continuous operators, an abstract kernel operator. We characterize the abstract kernel operators that map bounded sets onto precompact sets. In the process we generalize known characterizations of compact kernel operators and obtain some interesting new ones.

1. Preliminaries. Unless otherwise stated, all topological vector spaces encountered in this paper will be assumed to be Hausdorff. For notation and terminology concerning locally solid Riesz spaces not explained below, we refer the reader to [1]. For terminology concerning locally convex spaces we follow [14].

A linear topology τ on a Riesz space is said to be locally solid if τ has a neighborhood basis at zero consisting of solid sets. (A set V is said to be solid whenever $|x| \leqq |y|$ and $y \in V$ imply $x \in V$.) If τ is both locally convex and locally solid, then it is referred to as a locally convex-solid topology, and in this case, τ has a basis at zero consisting of solid and convex sets. The main properties connecting topological and order continuity needed for this work are the following:

1. *The Lebesgue property*: $u_\alpha \downarrow 0$ implies $u_\alpha \xrightarrow{\tau} 0$.
2. *The pre-Lebesgue property*: $0 \leqq u_n \uparrow \leqq u$ implies that $\{u_n\}$ is τ-Cauchy.