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COLLECTIONS OF COVERS OF METRIC SPACES

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In this paper cardinality κ collections of open covers of a topological space satisfying various conditions are studied. When $\kappa = \omega$ some of the conditions are equivalent to the space being metrizable and the union of a compact set and a discrete set. For a metrizable space some of the conditions are equivalent to complete metrizability. If $\kappa \neq \omega$ then the relationship between some of the conditions and the existence of scales is examined.

1. Introduction and definitions.

1.1. An ordinal number is the set of all ordinals which precede it and a cardinal number is an ordinal which cannot be put in a one-to-one correspondence with any ordinal which precedes it. Throughout this paper ω will denote the set of all finite ordinals and κ will denote an infinite cardinal number.

If M is a set, x is a point, and $\mathscr{H}$ is a collection of sets, then the star of M with respect to $\mathscr{H}$, denoted $st(M, \mathscr{H})$ is the union of all members of $\mathscr{H}$ which meet M and $st(x, \mathscr{H}) = st(\{x\}, \mathscr{H})$. A sequence $\mathscr{G} = \mathscr{G}_0, \mathscr{G}_1, \mathscr{G}_2, \cdots$ of open covers of a topological space S is called a development for S iff for each $x \in S$ and open set U containing x there is an n such that $st(x, \mathscr{G}_n) \subseteq U$. Moreover, a development is monotonic iff $\mathscr{G}_{n+1} \subseteq \mathscr{G}_n$ for all n. A space which admits a development is called a developable space and a regular-T_1 developable space is called a Moore space. A development $\mathscr{G}$ for a Moore space is star complete (see [16]) provided that if $\{M_0, M_1, M_2, \cdots\}$ is a sequence of closed sets such that for each n, $M_{n+1} \subseteq M_n \subseteq st(x, \mathscr{G}_n)$ for some $x \in S$ then $\bigcap M_n \neq \varnothing$. A Moore space having a star complete development is said to be star complete. A Moore space S is Moore-closed (see [5] and [6]) iff S is closed in each Moore space in which S is embedded.

A space S is a $w\Delta$-space (see [3]) iff there exists a sequence $\mathscr{B}_0, \mathscr{B}_1, \mathscr{B}_2, \cdots$ of open covers of S such that for each $x \in S$, if $x_n \in st(x, \mathscr{B}_n)$ then the sequence $\{x_0, x_1, x_2, \cdots\}$ has a cluster point. A space S has a G_δ^*-diagonal (see [10]) provided there is a sequence $\mathscr{G}_0, \mathscr{G}_1, \mathscr{G}_2, \cdots$ of open covers of S such that if x and y are distinct points of S, there is an n such that $y \notin \overline{st(x, \mathscr{G}_n)}$.

A nonempty subset M of a topological space S is called discrete iff for each $x \in M$ there is an open set U such that $U \cap M = \{x\}$. A collection of sets is discrete if the closures of the sets are mutually