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A SPECTRAL CONTAINMENT THEOREM ANALOGOUS TO THE SEMIGROUP THEORY RESULT $e^{t\sigma(A)} \subseteqq \sigma(e^{tA})$

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It is known that if A generates a (C_0) semigroup (e^{tA}), then $e^{t\sigma(A)} \subseteqq \sigma(e^{tA})$, where σ denotes "spectrum." This result is generalized to the context of solution operators of certain nth order linear differential equations.

1. Introduction. Let (T_t) be a (C_0) semigroup on a Banach space X with generator A. It is known [2, p. 457] that

$$e^{t\sigma(A)} \subseteqq \sigma(T_t) . \tag{1.1}$$

Since $T_t f$ solves the differential equation

$$\begin{cases} x' = Ax \\ x(0) = f , \end{cases} \tag{1.2}$$

$T_t f$ can be formally written as e^{tA}, so that (1.1) can be written as

$$e^{t\sigma(A)} \subseteqq \sigma(e^{tA}) . \tag{1.3}$$

In this paper it will be shown that if one replaces the first derivative in (1.2) by an nth order linear differential expression L producing the equation

$$\begin{cases} Lx = Ax \\ x(0) = f \\ x^{(j)}(0) = 0 \quad \text{for} \quad j = 2, \cdots, n-1 \end{cases}$$

solved by $S_t f$ for some linear operators S_t and replaces e^{tz}, the solution in C of

$$\begin{cases} x' = zx \\ x(0) = 1 , \end{cases}$$

by $\psi(t, z)$, the solution of

$$\begin{cases} Lx = zx \\ x(0) = 1 \\ x^{(j)}(0) = 0 \quad \text{for} \quad j = 2, \cdots, n-1 \end{cases}$$

(so that formally $S_t = \psi(t, A)$), then the analog

$$\psi(t, \sigma(A)) \subseteqq \sigma(S_t) ;$$
$$\psi(t, \sigma(A)) \subseteqq \sigma(\psi(t, A))$$