

# ON THE DECOMPOSITION OF REDUCIBLE PRINCIPAL SERIES REPRESENTATIONS OF $P$ -ADIC CHEVALLEY GROUPS

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**In this paper we study the decomposition of principal series representations of  $p$ -adic Chevalley groups which are induced from a minimal parabolic subgroup, and determine the structure of the commuting algebras of these representations.**

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**Introduction.** Let  $G$  be a split reductive  $p$ -adic group,  $T$  a maximal split torus of  $G$  and  $B = TU$  a minimal parabolic subgroup of  $G$ . A (unitary) character  $\lambda$  of  $T$  may be extended trivially across  $U$  to define a character of  $B$ . The induced representation  $\text{Ind}_B^G \lambda$  is called a (unitary) principal series representation of  $G$ .

Let  $W$  be the Weyl group of  $G$  and choose  $w \in W$ . Then the representations  $\text{Ind}_B^G \lambda$  and  $\text{Ind}_B^G w\lambda$  are equivalent. The problem of constructing explicit intertwining operators  $a(w, \lambda)$  between  $\text{Ind}_B^G \lambda$  and  $\text{Ind}_B^G w\lambda$  has been studied for real semi-simple Lie groups by Kunze and Stein [24, 25, 26] Schiffmann [30], Knapp [14, 15, 16] Knapp and Stein [17, 18, 19, 20, 21, 22] Harish-Chandra [10] and others. For groups defined over a  $p$ -adic field  $\mathfrak{k}$ , these operators were first studied for  $\text{SL}(2)$  by Sally [28], and then for  $p$ -adic Chevalley groups by Winarsky [36, 37], who used them to determine necessary and