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INTERPOLATION IN STRONGLY LOGMODULAR ALGEBRAS

RAHMAN YOUNIS

Let A be a strongly logmodular subalgebra of $C(X)$, where X is a totally disconnected compact Hausdorff space. For s a weak peak set for A, define $A_S = \{f \in C(X): f|_S \in A|_S\}$. We prove the following:

THEOREM 1. Let s be a weak peak set for A. If b is an inner function such that $b|_S$ is invertible in $A|_S$ then there exists a function F in $A \cap C(X)^{-1}$ such that $F = \bar{b}$ on s.

THEOREM 2. Let s be a weak peak set for A. If $U \in C(X)$, $|U| = 1$ on s and $\operatorname{dist}(U, A_S) < 1$, then there exists a unimodular function $\tilde{U}$ in $C(X)$ such that $\tilde{U} = U$ on s and $\operatorname{dist}(\tilde{U}, A) < 1$.

1. Introduction. The purpose of this paper is to prove certain properties related to strongly logmodular algebras.

In their study of Local Toeplitz operators, Clancey and Gosselin [3] established one of these properties in a special case (H^∞) under a highly restrictive condition. In [7], the author proved this property for H^∞ without any condition.

In the present paper, we obtain this and another property for arbitrary strongly logmodular algebras. The proofs in [3] and [7] use special properties of H^∞ that are not shared by arbitrary strongly logmodular algebra. In the present work we use new techniques.

Let A be a strongly logmodular subalgebra of $C(X)$, where X is a totally disconnected compact Hausdorff space. If s is a weak peak set for A, define $A_S = \{f \in C(X): f|_S \in A|_S\}$. The main results of this work are: Theorem 3.2. Let s be a weak peak set for A, and let b be an inner function such that $b|_S$ is invertible in $A|_S$. Then there exists a function F in $A \cap C(X)^{-1}$ such that $F = \bar{b}$ on s.

THEOREM 3.1. *Let s be a weak peak set for A, and let u be in $C(X)$ such that $|u| = 1$ on s and $\operatorname{dist}(u, A_S) < 1$. There exists a unimodular function $\tilde{u}$ in $C(X)$ such that $\tilde{u} = u$ on s and $\operatorname{dist}(\tilde{u}, A) < 1$.*

2. Preliminaries. Let X be a compact Hausdorff space. We denote by $C(X)[C_R(X)]$ the space of continuous complex [real] valued functions on X. The algebra $C(X)$ is a Banach algebra under the supremum norm $\|f\|_\infty = \sup\{|f(x)|: x \in X\}$.

Let A be a function subalgebra of $C(X)$. A subset S of X is