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MAPGERMS INFINITELY DETERMINED WITH RESPECT TO RIGHT-LEFT EQUIVALENCE

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Mather has given both algebraic and geometric characterizations of finitely determined germs. We conjecture analogous characterizations of infinitely determined germs and prove parts of this conjecture. Recall that two mapgerms f and g are (right-left) equivalent if there are germs of diffeomorphisms l and r such that $f = l \circ g \circ r$. A mapgerm f at x is finitely determined if there is a k such that every germ having the same k-jet as f at x is equivalent to f; f is infinitely determined if every germ having the same Taylor series at x as f is equivalent to f.

Let E_n denote the space of germs at 0 in R^n of C^∞ real valued functions and let m_n denote the unique maximal ideal in E_n. Let E_n^p denote the set of p-tuples of elements of E_n; m_n^k may denote k-tuples or may be the kth power of m_n—which should be clear from context. If f is analytic, let f_C denote its complexification.

THEOREM 1.1. (*Mather*; *see* [2] *and* [4]). *For f in m_n^p, the following are equivalent*:

(1) *f is finitely determined*;

(2) *$dfE_n^n + f^*E_p^p \supset m_n^k E_n^p$ for some $k > 0$*;

(3) *(assuming f analytic) f_C is locally multistable in a deleted neighborhood of* 0.

Since f_C is a germ, (3) is to be interpreted as saying that, for each representative F of the germ f_C, there is a deleted neighborhood U of 0 on which F is locally multistable (i.e., for each finite $S \subset U$ the germ of F at S is stable).

Since $dfm_n^n + f^*m_p^p$ represents the tangent space to the orbit of f, the motivation for (2) is clear (except for the replacement of m by E). Also clear is that the analogous condition for infinite determination is gotten by replacing k by ∞.

Gaffney (see [2]) explains the motivation for (3) as follows: "It is relatively easy to understand why a finitely determined germ should have this property. Any perturbation at zero, whose Taylor expansion vanishes to sufficiently high order there, can be removed by a coordinate change in source and target. However, by a suitable high order perturbation at zero, one can obtain any kind of a perturbation at some fixed x different from zero. This low order perturbation at x is also removed by the induced change in