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A SPECTRAL MAPPING THEOREM FOR LOCALLY COMPACT GROUPS OF OPERATORS

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If U is a suitably continuous representation of a locally compact abelian group G by means of isometries on a Banach space X, $\mu \to U(\mu)$ its extension to a representation of the convolution algebra $M(G)$ and $\mathrm{sp}(U)$ the spectrum of U, then the spectrum of $U(\mu)$ is not always equal to $\hat{\mu}(\mathrm{sp}(U))^-$, but it is so if the continuous part of μ is absolutely continuous.

1. Introduction. To be more explicit, given a representation U of G as above one forms a representation of $M(G)$, the Banach algebra of bounded regular measures on G, given by

$$U\colon \mu \in M(G) \longrightarrow U(\mu) = \int U(g)d\mu(g) \in B(X) .$$

In particular, if $G = \boldsymbol{R}$, $U(\mu)$ can be interpreted in a more classical way as a function of the infinitesimal generator $D = i(d/dg)U(g)|_{g=0}$ and denoted by $\hat{\mu}(D)$, where $\hat{\mu}$ is the Fourier transform of μ. Notice that in this case $\sigma(D) = \mathrm{sp}(U)$ [5, 9], where σ is the usual spectrum of the linear operator D and $\mathrm{sp}(U)$ is the spectrum of the representation U (see [2]).

Thus it is natural to study how far this functional calculus can be extended and a spectral mapping theorem holds. The setting of our study will be the algebra of local multipliers of $L^1(G)$.

If μ is a Dirac measure, A. Connes [3] proved that

$$\sigma(U(\mu)) = \hat{\mu}(\mathrm{sp}(U))^- .$$

Even if such a result does not always extend (we shall exhibit counterexamples) we prove it for the class of measures whose continuous part belongs to $L^1(G)$.

2. Statement of the main result. Let G be a locally compact abelian group; by a representation U of G on a Banach space X we mean a pointwise $\sigma(X, X^*)$-continuous homomorphism of G into the group of $\sigma(X, X^*)$-continuous isometries of X, where X^* is the dual of X or X is the dual of X^*. The case of bounded representations reduces to this one.

Let $M(G)$ be the Banach algebra of all bounded regular measures on G. Given any algebra $L^1(G) \subset M \subset M(G)$, we can form the representation of M induced by U: