PACIFIC JOURNAL OF MATHEMATICS
Vol. 103, No. 2, 1982

LOCALLY INVARIANT TOPOLOGIES ON FREE GROUPS

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In 1948, M. I. Graev proved that the free topological group on a completely regular Hausdorff space is Hausdorff, by showing that the free group admits a certain locally invariant Hausdorff group topology. In 1964, S. Świerczkowski gave a different proof, which also depends on the construction of a locally invariant topology. Yet another such construction follows from work of K. Bicknell and S. A. Morris. Graev's topology has proved to be essential in the investigation of free products of topological groups; Świerczkowski's topology is the key to the work of W. Taylor on varieties and homotopy laws; and Bicknell and Morris extend results of Abels on norms on free topological groups. In this paper, the three topologies are investigated in detail. It is seen that the Graev topology contains the Świerczkowski topology, which in turn contains that of Bicknell and Morris. These containments are shown to be proper in general. It is known that the topology of the free topological group is in general finer than each of these three topologies.

Introduction. If X is a completely regular Hausdorff space, let $F(X)$ denote the free group on the set X. Clearly the finest group topology on $F(X)$ which gives X its original topology must make $F(X)$ the free topological group on X. Because of this, a number of authors have constructed Hausdorff group topologies on $F(X)$ as a means of proving that the free topological group is Hausdorff. Moreover, most other proofs of this fact are easily seen to contain implicitly the construction of some group topology on $F(X)$.

In this paper we shall examine and compare the topologizations of $F(X)$ arising from three such constructions.

The topologies we study will all have the additional property of *local invariance*; that is, they have bases at the identity of sets invariant under inner automorphisms, or, equivalently, they are defined by families of invariant pseudometrics. (A pseudometric ρ on a group is (*two-sided*) *invariant* if $\rho(axb, ayb) = \rho(x, y)$ for all group elements a, b, x, y.) Such topologies arise naturally in the present context since they necessarily make the group operations continuous.