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# LOPSIDED SETS AND ORTHANT-INTERSECTION BY CONVEX SETS

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**Given a subset $L$ of the $2^d$ closed orthants in $d$-dimensional Euclidean space, is there a convex set $K$ which intersects those closed orthants in $L$, while missing those not in $L$? A strong combinatorial condition on $L$, which is necessary for the existence of such a convex set, is exhibited. This condition is studied and its close connections with the theory of oriented matroids are examined. The sets $L$ satisfying this condition — the "lopsided" sets — have a rich combinatorial structure which can be exploited in the study of convex sets and systems of linear inequalities.**

**1. Introduction.** Let $E$ be a finite set and denote by $V(E)$ the vector space of all real-valued functions $f$ on $E$. Let $C(E)$ denote the subset of $V(E)$ consisting of all functions $f$ from $E$ to the set $\{-1, 1\}$, so that $C(E)$ is the set of vertices of a cube in $V(E)$. If $f$ is in $C(E)$, then the set:

$$O(f) = \{g \in V(E) \colon g(e)f(e) \geq 0, \text{ for each } e \in E\}$$

is the closed orthant of $V(E)$ which contains $f$.

If $K$ is a convex subset of $V(E)$, let $L(K)$ denote the set:

$$\{f \in C(E) \colon K \cap O(f) \neq \varnothing\},$$

so that $f$ is in $L(K)$ if $K$ intersects the orthant of $V(E)$ corresponding to $f$. A subset $L$ of $C(E)$ will be called *realizable* if there is a convex set $K$ with $L = L(K)$. The realizable sets have in common a rather strong property, "lopsidedness," which will be described in §2. Examples showing how lopsided sets may arise in other settings will be given.

In §§3 and 4, two other descriptions of lopsided sets will be given. That of §4 is used in §5 to show how lopsided sets may be derived from oriented matroids. Also in §5, the property of lopsidedness is used to give a new description of the simple oriented matroids. (For a discussion of oriented matroids, see Folkman and Lawrence [**4**], or Bland and Las Vergnas [**1**].)

Finally, §6 gives an example of a lopsided set which is not realizable. This set is a subset of an 8-dimensional cube. It is not known whether this is the smallest dimension for such a set.