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THE PERTURBATION THEORY FOR LINEAR OPERATORS OF DISCRETE TYPE

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Using the theory of unconditional bases, we discuss the perturbation theory of linear operators of discrete type.

The principal abstract perturbation theorem about discrete spectral operators was introduced by J. T. Schwartz, and extended by H. P. Kramer to the general case ([**1**], XIX.2 Theorem 7). In this paper, we shall give a simple proof for Schwartz-Kramer's Theorem by using the theory of unconditional bases, and omit the condition of weak completeness in their theorem. In the proof of [**1**], XIX.2 Theorem 7, because of using [**1**], XVIII.2 Corollary 33, so that it needs the condition of weak completeness. On the other hand, all perturbant generalized eigenvectors consist of an unconditional basis, so we can prove the theorem without using the above corollary and omit the condition of weak completeness.

DEFINITION 1. A linear operator T in Banach space B is called discrete type ((D) type), if $\rho(T) \neq \varnothing$, and there exist an unconditional basis $\{x_n\}$ of B, a sequence of complex numbers $\{\lambda_n\}$ and a positive integer N, such that $\lim_n |\lambda_n| = +\infty$, $\lambda_n \neq \lambda_m$, $\forall\, n, m \in \mathbf{N}$, $m > N$ and $n \neq m$, $Tx_n = \lambda_n x_n$, $\forall\, n > N$, $T[x_1, \ldots, x_N] \subset [x_1, \ldots, x_N]$ and $\sigma(T \mid [x_1, \ldots, x_N]) = \{\lambda_1, \ldots, \lambda_N\}$.

PROPOSITION 2. *Let T be a linear operator of (D) type in Banach space B, $\{x_n\}$, $\{\lambda_n\}$ and N as in Definition* 1. *Then* $\sigma(T) = \{\lambda_n\}$,

$$\mathfrak{D}(T) = \left\{x \in B \mid \textit{if } x = \sum_n \alpha_n x_n, \textit{ then } \sum_{n>N} \lambda_n \alpha_n x_n \in B\right\}$$

$$Tx = \sum_{n=1}^{N} \alpha_n T x_n + \sum_{n>N} \lambda_n \alpha_n x_n, \qquad \forall\, x = \sum_n \alpha_n x_n \in \mathfrak{D}(T).$$

However, for each $\lambda \notin \sigma(T)$, $R(\lambda, T) = (T - \lambda I)^{-1}$ *is compact and*

$$R(\lambda, T)x = \sum_{n=1}^{N} \alpha_n (T - \lambda I)^{-1} x_n + \sum_{n>N} \frac{\alpha_n}{\lambda_n - \lambda} x_n,$$
$$\forall\, x = \sum_n \alpha_n x_n \in B.$$