PACIFIC JOURNAL OF MATHEMATICS
Vol. 104, No. 2, 1983

CRITERIA FOR OSCILLATORY SUBLINEAR SCHRÖDINGER EQUATIONS

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The semilinear Schrödinger equation

$$Lu \equiv \Delta u + f(x, u) = 0, \qquad x \in \Omega \tag{1}$$

will be considered in an exterior domain $\Omega \subset R^n$, $n \geq 2$, where f is nonnegative and locally Hölder continuous in $\Omega \times (0, \infty)$. One objective is to find sharp necessary conditions for (1) to be oscillatory in Ω under the *sublinear* hypothesis that $\max_{|x|=r} t^{-1}f(x, t)$ is a nonincreasing function of t in $(0, \infty)$ for each fixed $r > 0$. The necessary conditions below are proved in §2:

$$\int^{\infty} r \max_{|x|=r} f(x, c\log r)\, dr = +\infty \qquad \text{if } n = 2;$$

$$\int^{\infty} r \max_{|x|=r} f(x, c)\, dr = +\infty \qquad \text{if } n \geq 3$$

for some positive constant c. Sufficient conditions for (1) to be oscillatory in Ω are proved in §3 under a modified sublinear hypothesis. These results are then combined to yield characterizations of oscillatory sublinear equations of the Emden-Fowler type in exterior domains.

The sublinear Emden-Fowler (or Lane-Emden) equation is the prototype

$$\Delta u + p(x)\,|u|^{\gamma} \operatorname{sgn} u = 0, \qquad 0 < \gamma < 1, x \in \Omega, \tag{2}$$

where $p(x)$ is nonnegative and locally Hölder continuous in Ω. A theorem of Kitamura and Kusano [7] states in particular that (2) is oscillatory in a exterior domain Ω in R^n, $n \geq 2$, if

$$\int^{\infty} rP_1(r)\, dr = +\infty, \tag{3}$$

where $P_1(r) = \min_{|x|=r} p(x)$. The same is true if $P_1(r)$ is replaced by the spherical mean of $p(x)$ over the sphere of radius r (see §3). Under additional regularity hypotheses on $p(x)$ it was proved by E. S. Noussair and the writer [12] that (3) is in fact *necessary and sufficient* for (2) to be oscillatory in $\Omega \subset R^n$ if $n \geq 3$. However, this is not so if $n = 2$; an easy counterexample is provided in the case that (2) is a radial equation:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) + p(r)|u|^{\gamma} \operatorname{sgn} u = 0, \tag{4}$$