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FREE PRODUCTS IN THE CLASS OF ABELIAN l-GROUPS

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The main objective of this paper is to present several constructions of free products in the class of abelian l-groups which are sufficiently concrete to allow for an in depth examination of their structure. Some applications of these constructions are discussed, and it is shown that abelian l-group free products satisfy the subalgebra property. Further, some questions on free l-groups over group free products are considered for a variety of l-groups which is either abelian or contains the representable l-groups. Finally, a general observation is made about countable chains and countable disjoint sets in free algebras.

1. Introduction. Let $\mathcal{U}$ be a class of l-groups (lattice ordered groups) and $(G_i \mid i \in \mathcal{I})$ a family of members of $\mathcal{U}$. The $\mathcal{U}$-free product of this family is an l-group $G \in \mathcal{U}$, denoted by ${}^{\mathcal{U}}\bigsqcup_{i \in \mathcal{I}} G_i$, together with a family of l-monomorphisms $(\alpha_i\colon G_i \to G \mid i \in \mathcal{I})$ such that

(i) $\bigcup_{i \in \mathcal{I}} \alpha_i(G_i)$ generates G as an l-group;

(ii) for every $H \in \mathcal{U}$ and every family of l-homomorphisms $(\beta_i\colon G_i \to H \mid i \in \mathcal{I})$, there exists a (necessarily) unique l-homomorphism $\beta\colon G \to H$ satisfying $\beta_i = \beta\alpha_i$ for all $i \in \mathcal{I}$.

Following the usual practice we shall speak of ${}^{\mathcal{U}}\bigsqcup_{i \in \mathcal{I}} G_i$ as the $\mathcal{U}$-free product of $(G_i \mid i \in \mathcal{I})$. To simplify our notation, we use the "internal" definition of a $\mathcal{U}$-free product, that is, we identify each free factor G_i with its image $\alpha_i(G_i)$ in ${}^{\mathcal{U}}\bigsqcup_{i \in \mathcal{I}} G_i$, and thus we think of each G_i as an l-subgroup of ${}^{\mathcal{U}}\bigsqcup_{i \in \mathcal{I}} G_i$. As a consequence of general existence theorems (See Grätzer [**13**, p. 186] or Pierce [**25**, p. 107]), $\mathcal{U}$-free products always exist in any class of l-groups closed under products and l-subgroups.

In this paper we concentrate on the class $\mathcal{A}$ of abelian l-groups, although many of our results also hold in the important class of vector lattices. Our main goal is to develop a reasonable representation theory for $\mathcal{A}$-free products. This is done in §2 where we give several methods of constructing these products, among the most useful of which represents ${}^{\mathcal{A}}\bigsqcup_{i \in \mathcal{I}} G_i$ $(G_i \in \mathcal{A})$ as a subdirect product of totally ordered abelian groups each determined by the primes of the individual G_i's. We also show here how the $\mathcal{A}$-free products relate to the free abelian l-groups over partially ordered abelian groups.

The third and fourth sections of the paper are devoted to considering several different properties for free products of l-groups. In particular using the representation theory established in §2 we show that the subalgebra property is satisfied for $\mathcal{A}$-free products.