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DOUBLE TANGENT BALL EMBEDDINGS OF CURVES IN E^3

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An arc or curve J in E^3 has congruent double tangent balls if there exists a positive number δ such that for each $p \in J$, there are two three-dimensional balls B and B', each with radius δ, such that $\{p\} = B \cap B' = (B \cup B') \cap J$. Such an arc or simple closed curve is shown to be tamely embedded in E^3. An example is given to show that the "uniform" radii are required for this conclusion and to show the necessity of having two tangent balls at each point rather than just one. The proof applies as well to show that any subset of E^3 having these congruent double tangent balls must locally lie on a tame 2-sphere.

1. Introduction. Questions about tameness of a 2-sphere in E^3 when the sphere has double tangent balls apparently originated with R. H. Bing [**1**]. Bothe [**2**] and I [**7**] independently showed that such a 2-sphere is tamely embedded in E^3, and, more recently Daverman, Wright and I [**4**], [**8**] showed the existence of wild $(n-1)$-spheres in E^n having double tangent balls for each $n > 3$.

Let J be a subset of E^3, and let $p \in J$. Then J is said to have δ *double tangent balls* at p provided there exist a positive number δ and two 3-dimensional balls B and B' each of radius δ such that $B \cap B' = \{p\} = (B \cup B') \cap J$. If there exists a positive δ such that J has these δ double tangent balls at each of its points, then J is said to have *congruent double tangent balls*. When J is a 2-sphere one may also require that the interiors of the double tangent balls lie in different components of $E^3 - J$, as was done in the previous studies [**2**], [**3**], [**4**], [**6**], [**7**], [**8**]. However this paper concentrates on curves in E^3 where no such restriction can be imposed because the curves are not assumed to be subsets of spheres. It should also be noted that the global uniformity (congruence) of the tangent balls over J was not part of Bing's question, although Griffith [**6**] did answer the question for 2-spheres in E^3 with this extra hypothesis. In fact, an $(n-1)$-sphere in E^n is tame when it has congruent double tangent balls on opposite sides of the sphere at each of its points [**4**].

It is natural to wonder whether an arc J in E^3 is tame with the weaker hypothesis that it have double (but not necessarily congruent) tangent balls or that it have congruent (but not necessarily double) balls tangent to J at each of its points. After all, either of these two weaker conditions implies the tameness of a 2-sphere in E^3 ([**2**], [**7**], [**5**]), provided the balls are in the appropriate complementary domains of the sphere. Wild arcs in E^3 having double tangent balls at each of their points are easy to construct from known examples. Such an arc F can be constructed to also