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ULTRAFILTERS AND MAPPINGS

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We give characterizations of closed, quasi-perfect, *d*-, *Z*-, *WZ*-, *W-open, *N*-, *WN*-, W_rN- and other maps using closed or open ultrafilters and investigate relations between these maps and various properties as generalizations of realcompactness, i.e., almost-, *a*-, *c*- and *wa*-real compactness, *cb**-ness and weak *cb**-ness. Finally we establish several theorems about the perfect *W**-open image of a weak *cb** space and its application to the absolute $E(X)$ of a given space *X*.**

We characterize closed, *Z*-, *WZ*-, *N*- and *WN*-maps by closed ultrafilters in §1 and show that φ is W^*-open iff $\varphi^{\#}\mathfrak{U}$ is an open ultrafilter for each open ultrafilter $\mathfrak{U}$ in §2. In §3, introducing the notion of *-open map, we show that $\beta\varphi$ is open iff φ is a *-open W_rN-map iff there is $\mathfrak{U}^p$ with $\varphi^{\#}\mathfrak{U}^p = \mathfrak{V}^q$ for each $q \in \beta Y$, each $\mathfrak{V}^q$ and each $p \in (\beta\varphi)^{-1}q$. In §4, we discuss invariance concerning CIP of closed or open ultrafilters under various maps and establish invariances and inverse invariance of various properties as a generalization of realcompactness under suitable maps in §5. In §6, we give several theorems about the perfect W^*-open image of weak cb^* spaces which contain, as corollaries, known results concerning the absolute $E(X)$ of X.

Throughout this paper, by a space we mean a completely regular Hausdorff space and assume familiarity with [**3**] whose notion and terminology will be used throughout. We denote by $\varphi: X \to Y$ a continuous onto map and by $\beta X(\upsilon X)$ the Stone-Čech compactification (realcompactification) of X and by $\beta\varphi$ the Stone extension over βX of φ. In the sequel, we use the following notation and abbreviation. $N =$ the set of positive integers, CIP = countable intersection property, nbd = neighborhood, $\mathfrak{F}^p =$ a closed ultrafilter converging to p. We denote by $\mathfrak{F}(\mathfrak{U})$ a closed (open) ultrafilter on X and by $\mathfrak{S}(\mathfrak{V})$ a closed (open) ultrafilter on Y. $\varphi^{\#}\mathfrak{F} = \{E \subset Y;\ \varphi^{-1}E \in \mathfrak{F}$ and E is closed in $Y\}$. Similarly define $\varphi^{\#}\mathfrak{U}$.

1. Closed ultrafilters.

1.1. In the sequel, we use frequently the following results.

(1) *If* $p \in \bigcap \mathrm{cl}_{\beta X}\varphi^{-1}\mathfrak{S}^q = \bigcap\{\mathrm{cl}_{\beta X}\varphi^{-1}E;\ E \in \mathfrak{S}^q\}$, *then there is* $\mathfrak{F}^p$ *with* $\varphi^{\#}\mathfrak{F}^p = \mathfrak{S}^q$. For, $\mathfrak{C} = \{\varphi^{-1}E \cap F;\ E \in \mathfrak{S}^q,\ F \in N(p)\}$ is a closed filter base where $N(p)$ is a closed nbd base of p in βX. Obviously $\mathfrak{C} \to p$. Thus any $\mathfrak{F}^p$ containing $\mathfrak{C}$ has the property $\varphi^{\#}\mathfrak{F}^p = \mathfrak{S}^q$. It is easily seen that the same method above can be applied to open ultrafilter and