PACIFIC JOURNAL OF MATHEMATICS
Vol. 104, No. 2, 1983

WORD PROBLEMS FOR FREE OBJECTS IN CERTAIN VARIETIES OF COMPLETELY REGULAR SEMIGROUPS

J. A. GERHARD AND MARIO PETRICH

Semigroups which are unions of groups are said to be completely regular. They form a variety when considered as semigroups with an operation of inverse. The variety of bands and the variety of completely simple semigroups are subvarieties. The present paper reduces the word problem for the free semigroups in each subvariety $\mathcal{V}$ of the join of the variety of bands and the variety of completely simple semigroups to the word problem for certain groups in $\mathcal{V}$. In particular if the word problems for the latter have a solution so does the word problem in $\mathcal{V}$.

Semigroups which are unions of groups are said to be *completely regular*. A completely regular semigroup S is provided in a natural way with a unary operation of inverse by defining a^{-1} for $a \in S$ to be the group inverse of a in the maximal subgroup of S to which a belongs. This operation satisfies the identities

$$(1) \qquad xx^{-1}x = x,$$

$$(2) \qquad xx^{-1} = x^{-1}x,$$

$$(3) \qquad (x^{-1})^{-1} = x.$$

In fact a completely regular semigroup can be defined as a semigroup with a unary operation which satisfies these identities. The class of completely regular semigroups is therefore a variety of (universal) algebras with one unary and one binary operation. This variety is denoted by $\mathcal{CR}$. Two important subvarieties are the variety of bands $\mathcal{B}$ and the variety of completely simple semigroups $\mathcal{CS}$. The join of these two varieties has recently been described in [4] and [9]. It is the subvariety of $\mathcal{CR}$ defined by the identity

$$(xy)^0x^0(zx)^0 = (xyxzx)^0,$$

where as usual $u^0 = uu^{-1}$.

In [4] this variety is called the variety of pseudo orthodox bands of groups. Following the terminology of [6] it might be called the variety of pseudo orthocrypto groups. Here we will simply refer to it as $\mathcal{B} \vee \mathcal{CS}$.

The present paper gives a solution to the word problem for the free semigroups in each of the subvarieties of $\mathcal{B} \vee \mathcal{CS}$. A solution for the subvarieties of $\mathcal{B}$ is known (see [3]). The free objects in $\mathcal{CS}$ have been