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CLOSED IDEALS OF $l^1(\omega_n)$ WHEN $\{\omega_n\}$ IS STAR-SHAPED

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Let $A = l^1(\omega_n)$ be a radical Banach algebra of power series where the weight $\{\omega_n\}$ is star-shaped. Let T be the operator of right translation on A. We give sufficient conditions for all closed ideals of A to be standard. These cases are more general than those previously considered, since in all these cases, T is unicellular but not a basis operator. We also construct a large class of such algebras A in which there are elements x such that the closed ideal $(Ax)^-$ is standard, but the algebraic ideal Ax contains no power of z.

1. Introduction. In this paper we study algebras $A = l^1(\omega_n)$ where

$$l^1(\omega_n) = \left\{ \sum_{n=0}^{\infty} \alpha_n z^n : \sum_{n=0}^{\infty} |\alpha_n| \omega_n < \infty \right\}.$$

We shall be concerned entirely with the case when $\{\omega_n\}$ is a star-shaped weight, i.e. essentially that the region below the graph of $\ln \omega_n$ is illuminated by the origin (see Definition 2.1). For these weights A is a radical Banach algebra of power series with unit adjoined, although in the following we shall refer to these algebras simply as radical Banach algebras. The multiplication is, of course, the usual multiplication of formal power series. There are obvious closed ideals in $A = l^1(\omega_n)$, the so called *standard* ideals:

$$K(\infty) \equiv \{0\}$$

and

$$K(n) \equiv \left\{ \sum_{j=n}^{\infty} \alpha_j z^j \in A \right\}, \qquad n = 0, 1, 2, \ldots.$$

Any other closed ideals are referred to as *non-standard* ideals. At present it is not known whether there are *any* weights $\{\omega_n\}$ such that $l^1(\omega_n)$ is a radical Banach algebra and contains a non-standard ideal. So called *Schauder type* ideals, which would have to be non-standard, have been conjectured to exist and an erroneous construction [**5**, p. 205] appears in the literature (see [**7**, 2. Schauder Type Ideals] for a specific discussion of the error). We note that if one removes the restriction that $l^1(\omega_n)$ be an *algebra*, examples can be given where the right shift operator on The Banach space $l^1(\omega_n)$ is quasinilpotent and has non-standard closed invariant subspaces [**6**]. In these examples $l^1(\omega_n)$ is very far from being an