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VECTOR EXTENSIONS OF OPERATORS IN L^p SPACES

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We study B-valued extensions of operators of weak or strong type (p, q), where B is a p-Banach space of a certain type, and present several applications.

An old well known result of Marcinkiewicz and Zygmund states that every bounded linear operator T from L^p to L^q has a bounded extension $T \otimes 1_B$ from L^p_B to L^q_B, where B is a Hilbert space. The analogous question for certain types of Banach spaces was also considered in **[9]** and **[5]** and, for weak type operators, in **[13]** and **[14]**. Here we obtain a general result on the extension of bounded linear operators of (weak or strong) type (p, q) to B-valued functions, where B belongs to C. Herz's class of r-spaces (see **[5]**). Applications are given to pointwise convergence of vector valued functions and to weighted norm inequalities for a wide class of operators. Finally, we prove a mixed norm estimate for translation invariant operators which is a weak type analogue of the main result in **[6]**.

I. B-Valued extensions of operators. Let (X, μ) and (Y, ν) be measure spaces. Given a linear operator T from $L^p(\mu)$ to $L^0(\nu) = \{$all ν-measurable functions with the topology of local convergence in measure$\}$, and a quasi-Banach space B, the operator

$$T^B = T \otimes 1_B \colon \sum b_i f_i(x) \mapsto \sum b_i T f_i(y) \qquad (b_i \in B; f_i \in L^p(\mu))$$

is defined a priori on $L^p(\mu) \otimes B$. If T is of weak or strong type (p, q), or simply continuous in measure, we ask ourselves if the corresponding continuity condition holds for T^B, in which case, it can be uniquely extended to $L^p_B(\mu)$. In this case, and when B is a Banach space, T^B is characterized by the property:

$$(1) \qquad \langle T^B f(y), b' \rangle = T(\langle f, b' \rangle)(y) \quad \nu\text{-a.e.}$$

for every $f \in L^p_B(\mu)$ and $b' \in B'$ (dual of B).

By $\mathfrak{B}_r$, $0 < r < \infty$, we denote the class of all quasi-Banach spaces which are isomorphic to some subspace of a quotient of a space L^r. The following facts are either known or easy to check:

(F.1) When $r \leq 1$, $B \in \mathfrak{B}_r$ if and only if B is r-normed, since every r-Banach space is isomorphic to a quotient of $l^r(I)$ (Shapiro **[15]**).