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SUPER-PRIMITIVE ELEMENTS

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Given an extension, $R \subseteq T$, of commutative integral domains with identity, we say an element $u \in T$ is super-primitive over R, if u is the root of a polynomial $f \in R[x]$ with $c_R(f)^{-1} = R$, i.e., a super-primitive polynomial. The main purpose of this paper is to provide "super-primitive" analogues to some work of Gilmer-Hoffmann and Dobbs concerning primitive elements. (An element $u \in T$ is called primitive over R, if u is the root of a polynomial $f \in R[x]$ with $c_R(f) = R$.)

1. Introduction. Given an extension, $R \subseteq T$, of commutative integral domains with identity, we say an element $u \in T$ is super-primitive over R, if u is the root of a polynomial $f \in R[x]$ with $c_R(f)^{-1} = R$, i.e., a super-primitive polynomial. By $c_R(f)$, we mean the ideal of R generated by the coefficients of f, and when no confusion may result, we will write $c(f)$. Our primary motivation for investigating super-primitive elements is some work of Gilmer and Hoffmann [**6**], and some extensions of that work by Dobbs [**4**]. In particular, their studies dealt with, in the terminology of [**4**], primitive elements. An element $u \in T$ is said to be primitive over R, if u is a root of a polynomial $f \in R[x]$ with $c(f) = R$. It is shown [**4**, Theorem] (in the more general context of commutative rings with identity) that u is primitive over R if and only if $R \subseteq R[u]$ satisfies INC (incomparability). The main purpose of this paper is to consider a natural super-primitive analogue for this result, and to indicate some interesting related ideas.

Throughout this paper, all rings considered will be domains, i.e., commutative integral domains with identity, and any unexplained terminology is standard as in [**5**] or [**11**]. It should be noted that several of the results in these pages could be stated in the generality of commutative rings with identity, however we feel the main thrust of our work lies within the category of domains.

2. Super-primitive elements and associated primes of principal ideals. Let $\mathcal{P}(R) = \{P \in \operatorname{Spec}(R)\colon P$ is minimal over $(a : b)$ for some $a, b \in R\}$. The elements of $\mathcal{P}(R)$ are referred to as the associated primes of principal ideals [**2**]. A useful result concerning $\mathcal{P}(R)$, which we shall employ frequently, is due to Tang [**15**, Theorem E], and is stated as follows: (a) For a finitely generated ideal I of R, $I \subseteq P$ for some $P \in \mathcal{P}(R)$ if and only if $I^{-1} \neq R$; and, (b) $R = \bigcap_{P \in \mathcal{P}(R)} R_P$. It is immediate from this