PACIFIC JOURNAL OF MATHEMATICS
Vol. 105, No. 1, 1983

ON EQUIVALENT CATEGORY BASES

JOHN. C. MORGAN II

Any category base in which every region contains a minimal region is equivalent to a topology.

Utilizing the notion of a category base, which is a generalization of the notion of a topology, the author has developed a general theory of point sets within which many of the analogies between Lebesgue measure and Baire category have been unified (cf. [**3**]–[**9**]). In view of the "equivalence" between the Lebesgue measurable sets and the sets having the Baire property relative to the density topology (cf. [**10**, Chapter 22]), the question arises whether every category base is equivalent to some topology. We do not know the answer to this question. However, we shall show in this article that for a certain class of category bases, including all finite category bases, there do exist equivalent topologies.

After stating pertinent facts in §1, we define in §2 a basic topology which is associated with a given category base. In §3 we determine this basic topology in several examples and see that it is not in general equivalent to the category base from which it arises. As we show in §4, however, any category base in which every region contains a minimal region is equivalent to its basic topology.

1. Preliminaries. In this section we recall relevant definitions and theorems which were established in [**4**].

DEFINITION. A pair $(X, \mathcal{C})$, where $\mathcal{C}$ is a family of subsets of a nonempty set X, is called a category base if the nonempty sets in $\mathcal{C}$, called regions, satisfy the following axioms:

1. Every point of X belongs to some region; i.e. $X = \bigcup \mathcal{C}$.

2. Let A be a region and let $\mathfrak{D}$ be any nonempty family of disjoint regions which has power less than the power of $\mathcal{C}$.

(a) If $A \cap (\bigcup \mathfrak{D})$ contains a region then there is a region $D \in \mathfrak{D}$ such that $A \cap D$ contains a region.

(b) If $A \cap (\bigcup \mathfrak{D})$ contains no region then there is a region $B \subset A$ which is disjoint from every region in $\mathfrak{D}$.

It is readily seen that every topology is a category base.

With respect to a given category base we have the following generalization of the topological notions of nowhere dense sets, sets of the first category, and sets of the second category.