

ON A NEW TYPE OF L -FUNCTION FOR ALGEBRAIC CURVES OVER FINITE FIELDS

DAVID GOSS

Recently there has emerged a new theory of curves over a finite field. This theory is exciting in that it establishes previously unknown analogies between cyclotomic fields and function fields over a finite field. An extremely important aspect of this work is a new type of L -series for these function fields. These L -series bear quite remarkable and exciting similarities to classical L -series of number fields. The purpose of this paper is to describe these new L -series in detail.

TABLE OF CONTENTS

INTRODUCTION	143
1. BACKGROUND	145
2. THE THEORY AT INFINITY	148
2.1 Basic concepts	
2.2 A remark on the values of Dirichlet series	
2.3 Definition of π -adic functions and their analytic continuation	
2.4 Complements	
3. DESCRIPTION OF VALUES AT POSITIVE INTEGER POWERS	156
3.1 The Γ -ideal	
3.2 Special values of zeta functions at positive integral powers $\equiv 0(r-1)$	
3.3 Special values of partial zeta-functions at positive integral powers	
4. THE RELATIVE ZETA-FUNCTIONS	161
5. THE THEORY WHEN s_1 IS A NEGATIVE INTEGER	163
5.1 The algebraicity of L -series for $s_1 = -i$	
5.2 Some trivial zeros for zeta and L -functions	
5.3 Some additional results when $A = \mathbb{F}_r[T]$	
5.4 The v -adic theory and interpolation	
5.5 A remark on functional equations	
6. THE CONNECTION WITH DISTRIBUTION THEORY	175
6.1 Distribution and measures	
6.2 The π -adic theory	
6.3 Group theoretic interpretation of zeros	
6.4 The v -adic theory	
7. GALOIS GROUP INTERPRETATION VIA CLASS-FIELD THEORY	179
REFERENCES	181

Our desire to study such functions arose out of the following fact (discovered by L. Carlitz and independently, but much later, by the author): Let $i \in \mathbb{N}^+$. Then there exists a non-zero constant λ such that the convergent $1/T$ -adic sum

$$\lambda^{-i} \sum_{0 \neq \beta \in \mathbb{F}_r[T]} \beta^{-i}, \quad i \equiv 0(r-1),$$