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QUASI-REGULAR NEARNESS SPACES AND EXTENSIONS OF NEARNESS-PRESERVING MAPS

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Every basic nearness (or quasi-nearness) induces a Čech closure operator. There is a 1-1 correspondence between the cluster generated Riesz nearnesses on a given T_1 closure space and the principal (or strict) T_1 extensions of the space. (In particular linkage compact extensions correspond to proximal nearnesses, F-linkage compact extensions correspond to contigual nearnesses, while ordinary compact extensions correspond to cluster generated weakly contigual nearnesses.

In this paper we discuss conditions under which a nearness-preserving map between nearness spaces can be extended to a continuous map between the corresponding principal extensions of the induced closure spaces. The concept of a quasi-regular nearness space plays an important role in this connection. The general results on extensions of nearness-preserving maps are used to obtain results on extension of continuous maps into regular linkage compact and F-linkage compact spaces.

1. Introduction. Nearnesses on a topological space can be used as a means of introducing extensions of the space. Work of Bentley [**1**], Bentley and Herrlich [**2**], Herrlich [**8**], Naimpally [**11**] and Reed [**12**] show that every Lodato nearness on a given T_1 topological space gives rise to a principal (or strict) T_1 extension of the space, and all the principal T_1 extensions can be obtained in this way. (Compact extensions correspond to contigual nearnesses and linkage compact (or clan complete) extensions correspond to proximal nearnesses.) A nearness-preserving map between nearness spaces can be extended to a continuous map between the corresponding extensions of the underlying topological space, if suitable regularity conditions are imposed on the image space.

Basic nearnesses (quasi-nearnesses in the terminology of Herrlich [**8**]) induce closure spaces rather than topological spaces. It was recently shown by Chattopadhyay, Njåstad and Thron [**4**] that the above correspondence between Lodato nearnesses and principal extensions can be extended: Every Riesz nearness on a given T_1 closure space gives rise to a principal T_1 extension of the space, and all the principal T_1 extensions can be obtained in this way. (Linkage compact extensions correspond to proximal nearnesses, F-linkage compact extensions correspond to contigual nearnesses, while compact extensions correspond to weakly contigual cluster generated nearnesses.)

In the present paper we discuss conditions under which a nearness-preserving map between (basic) nearness spaces can be extended to a continuous map between the corresponding principal extensions of the