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REGULARITY OF THE BERGMAN PROJECTION IN CERTAIN NON-PSEUDOCONVEX DOMAINS

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Suppose D is a smooth bounded domain contained in $\mathbf{C}^n$ ($n \geq 2$) whose Bergman projection satisfies global regularity estimates, and suppose K is a compact subset of D such that $D - K$ is connected. The purpose of this note is to prove that, under these circumstances, the Bergman projection associated to the domain $D - K$ satisfies global regularity estimates.

This result is presently known only in very special cases when both D and K have a particularly simple form. For example, the fundamental paper of Kohn [5] reveals that if Ω_1 and Ω_2 are two smooth bounded strictly pseudoconvex domains in $\mathbf{C}^n$ ($n > 2$) such that $\Omega_2 \subset\subset \Omega_1$, then the $\bar{\partial}$-Neumann problem for the domain $\Omega_1 - \overline{\Omega}_2$ is subelliptic. Kohn's formula, $P = I - \bar{\partial}^* N \bar{\partial}$, which relates the Bergman projection P to the $\bar{\partial}$-Neumann operator N, shows that the Bergman projection associated to $\Omega_1 - \overline{\Omega}_2$ satisfies global regularity estimates. Recently, Derridj and Fornaess [3] have shown that if Ω_1 and Ω_2 are two pseudoconvex domains with real analytic boundaries in $\mathbf{C}^n$ with $n \geq 3$ and $\Omega_2 \subset\subset \Omega_1$, then the $\bar{\partial}$-Neumann operator for $\Omega_1 - \overline{\Omega}_2$ satisfies subelliptic estimates. Hence, the Bergman projection associated to $\Omega_1 - \overline{\Omega}_2$ satisfies global estimates in this case, also.

In Bell and Boas [2], it is proved that the Bergman projection associated to a smooth bounded complete Reinhardt domain satisfies global regularity estimates. Thus, there are more subtle examples of non-pseudoconvex domains for which regularity of the Bergman projection holds than those addressed by the theorem of the present work. Recently, the techniques used in [2] have been refined by David E. Barrett [1] to prove that the Bergman projection associated to a smooth bounded domain with a Lie group of transverse symmetries satisfies global regularity estimates.

The question as to whether or not the Bergman projection associated to a domain satisfies global regularity estimates is very important in problems relating to boundary behavior of holomorphic mappings (see [2]).

The Bergman projection P associated to a bounded domain D contained in $\mathbf{C}^n$ is the orthogonal projection of $L^2(D)$ onto $H(D)$, the closed subspace of $L^2(D)$ consisting of L^2 holomorphic functions. The space $C^\infty(\overline{D})$ is defined to be the set of functions in $C^\infty(D)$, all of whose